Polarization Calibration of VLBI Data

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ABSTRACT

This document discusses several techniques for the calibration of the polarized response of radio interferometers. Special attention is paid to the problems of Very Long Baseline Interferometers but the results are applicable to short baseline interferometers. A linearized model of feed response and the ellipticity – orientation model are discussed. The interaction between phase calibration and polarization calibration is considered as is the effect of spatial resolution of the polarization calibrators. A suggested calibration procedure is outlined.

1 Introduction

The purpose of this document is to describe various aspects of the calibration of interferometric data sensitive to the structure of radio sources in polarized light. This discussion follows the development of Cotton 1989. A good discussion of polarization in the context of radio interferometry is given in Fomalont and Wright 1974. The methods described in this document are implemented in the NRAO AIPS data analysis package as an option in the standard polarization calibration; most notably the task PCAL.

Polarization sensitive measurements are usually (but not always) made by cross correlating signals from pairs of detectors sensitive to orthogonal polarizations. For historical reasons these detectors will be referred to as feeds. In the following, the discussion will be limited to the case of feeds nominally sensitive to right and left circular polarization. In this case, the relationship between the measured correlations and the correlations of the Stokes' i, q, u and v values are:

$$i = 0.5 \times (RR + LL)$$
$$q = 0.5 \times (RL + LR)$$
$$u = 0.5 \times \sqrt{-1}(LR - RL)$$
$$v = 0.5 \times (RR - LL)$$

RR, LL, RL, and LR represent the correlations derived from the various combinations of right (R) and left (L) circular feeds. RR and LL will be referred to as parallel polarized correlations and RL and LR as cross polarized. I, Q, U and V will be used to represent the Stokes' parameters of the derived image. Alternately, the observed correlations are related to the Stokes' parameters by:

$$RR = i + v$$
$$LL = i - v$$
$$RL = q + \sqrt{-1}u$$
$$LR = q - \sqrt{-1}u$$

The calibration of polarization sensitive data involves two distinct steps: 1) the determination of the true instrumental feed response and the correction of the data to what would have been observed with perfect feeds and 2) correction of the apparent polarization angle (the angle of the apparent E-vectors of the polarized radiation on the sky) to the correct value at the top of the atmosphere.

The first step is necessary because any feed will have imperfections, i.e. will respond to signals with polarizations other that the intended one. There are two basic approaches to this problem, 1) model the feed as sensitive to elliptical radiation or 2) model the feed as sensitive to the desired polarization plus a complex factor times the orthogonal polarization. The first approach is relatively general but is strongly non linear and therefore computationally expensive. The second method can be easily linearized and therefore cheaper to compute but in this case is adequate only for nearly perfect feeds.

Calibration of the polarization angle is necessary for a number of reasons. The usual phase calibration schemes adjust the systems of parallel hand feeds to internally consistent values but allow an arbitrary phase difference between the two parallel hand systems. This converts directly into an arbitrary, but constant, rotation of the apparent orientation of the E-vectors on the sky. The other problem is that the propagation of the signal through the magnetized plasma of the ionosphere will cause an apparent rotation of the E-vectors. This effect, when significant, can be quite variable with time and observing geometry.

2 Interaction with Total Intensity Calibration

In practice, the response of an interferometer is modified by the earth's atmosphere and the instrument itself. The effects on amplitude, phase, etc. should be determined simultaneously with the parameters needed to describe the response to polarized signals. However, if the feeds are nominally sensitive to right and left circular polarization and weakly circularly polarized calibrator sources are used, then amplitude, phase, delay and rate corrections can be determined from the measurements obtained using the parallel polarized systems of feeds and applied to the data before determining the polarization parameters. This method will break down if the feeds depart too much from circular or the calibrators have significantly circular polarization. Fortunately, these conditions are met for a large range of circumstances.

If amplitude and phase calibration is applied before determining the polarization parameters the details of the calibration process will affect the detailed model of the response to a polarized signal.

2.1 Phase

The best example of the interaction between phase calibration and polarization calibration is the effect of parallactic angle. Fixed feeds on an antenna with an altitude-azimuth (alt-az) mount will appear to rotate with time as viewed from the source. This apparent orientation of the feed is called the parallactic angle. For circular feeds the effect of the parallactic angle on the parallel polarized correlations is to rotate the phases by the difference in the parallactic angles of the two antennas involved. Cross polarized correlations have their phases rotated by the sum of the parallactic angles. If a correction for the parallactic angle is not made to the data before phase calibration, the phase calibration process will include the parallactic angle of the phase reference antenna in the calibration phases. Since the effect of parallactic angle is different for parallel and cross polarized data, the parallactic angle of the reference antenna must be known to determine and apply corrections for the instrumental polarization. Since the parallactic angle of the reference antenna, or even the reference antenna itself, is a function of time, this leads to a messy bookkeeping problem. An alternate approach to phase calibration is to correct the phases of the data for the effects of parallactic angle before phase calibration. Since this correction is a function only of geometry it is easily computed. In either of these cases the model used to determine to polarization characteristics must reflect what has been done about the parallactic angles.

A similar problem arises with modeling the feeds as sensitive to elliptically polarized signals. In this case the phases of the correlations are rotated by the orientation of the feed of the reference antenna in a manner similar to that for parallactic angle. The principle differences are that this angle is constant in time but unknown prior to calibration. The difficulty here is that the same reference antenna may not be used for all observations; a common situation in VLBI observations is that no antenna participates in all observations. In this case, either the phase calibration of all the data must be referred to the same reference antenna, even at times when it was not used, or the reference antenna used for each datum must be known.

During phase calibration the phases must all be referred to the same antenna for another reason. In the calibration of the orientation of the E-vectors it is usually assumed that the two orthogonal phases systems are internally consistent and only differ by a constant phase (ignoring the possibility of ionospheric Faraday rotation). This will only be true if the phases are all referred to the same antenna and thus the phase difference between the orthogonal systems is the one for that antenna. This may require referencing phases etc. to an antenna at times for which it was not observing the source.

The bookkeeping problems in polarization calibration are greatly simplified if the phases of the data are corrected for parallactic angle before phase calibration. All following discussion will assume that this has been done and that all phases are referred to the same reference antenna.

2.2 Delays and Rates

Since delays and rates are simply the frequency and time derivatives of phase, calibration of residual delay and rate errors will be similar to that of residual phase errors. Since the right and left hand systems involve substantially different electronics and cabling, there may be a delay difference between these two systems. Properly functioning phase cal measurements will remove these differences. However, the phase cals used with MkIII observations cannot correct single-band delays as they only involve no more that a single tone per IF. If no phase cals are available then there may be an offset in the multi-band delays and in each of the single-band delays between the right and left hand systems. Since the usual method of residual delay error calibration is to measure the differences from a reference antenna, the derived delay corrections must all be referred to the same antenna for the entire dataset if there is to be a constant set of delay residual offsets between the two systems. If there is a constant set of delay residual offsets for the data set then these offsets can be estimated from a single calibrator observation on a single baseline.

The antenna electronics should be sufficiently stable that the time derivatives of any right-left differences, i.e. rates, should be very small. If this is not the case then the data is not sufficiently coherent to be calibrated. The following will ignore any right-left rate differences.

2.3 Amplitude

Amplitude calibration, if done prior to polarization calibration, may also affect the polarization calibration. The usual assumption in the amplitude calibration stage is that the calibrators have no circular polarization and the feeds are perfectly circularly polarized. Errors in these assumptions will cause the addition of vectors to the measured correlations. Since these errors are additive they will not factorize into antenna components and will appear partly as baseline dependent amplitude errors. If these errors are small they may be ignored. If they are large then several iterations of amplitude and polarization calibration using the ellipticity-orientation model may be necessary.

3 Source Resolution

One of the many complications of polarization calibration is that the very compact sources suitable for phase and polarization calibration are frequently variable in both total intensity and polarized emission. This means that the polarization of the calibrators must be determined simultaneously with the instrumental polarization. Fortunately, alt-az mounts cause the contribution to the measured correlations due to the polarized emission from the source to rotate with parallactic angle whereas the instrumental contribution is constant. Hence, if observations are made with a range of parallactic angle, source polarization may be separated from instrumental polarization.

This process is simple enough if the source is spatially unresolved; a value of Q and U for the source is sufficient. If the source is resolved then the spatial distribution of the polarized emission must be known or determined. At typical VLBI resolutions there are no completely unresolved calibrators and this problem becomes fairly serious.

One way of relaxing the requirement that the calibrator be unresolved is to assume that the polarized emission is a scaled version of the total intensity with a constant orientation of the E-vectors. In this case, the only free parameters are the source fractional Q and U. Unfortunately, this is usually a rather poor assumption; the peak polarized emission is frequently not coincident with the peak total intensity and the orientation of the E-vectors may vary dramatically. However, for a completely unpolarized calibrator this approximation is completely adequate and most potential calibrators are weakly polarized. In this case it is possible to initially solve for the source plus polarization parameters, image the calibrator in polarized emission and then iteratively use the deconvolved model to refine the determination of the instrumental polarization parameters.

4 Models of the Feed Response

The response of the feeds is usually represented by a model characterized by a small number of parameters. Two such models are discussed in this section.

4.1 Ellipticity - Orientation Model

A general model for the feed response is to assume that the feed responds to elliptical polarization and is characterized by the ellipticity and orientation of the ellipse. Following Fomalont and Wright 1974 we will parameterize the response of a feed to the electric field as:

$$\mathbf{G} = \mathbf{e}_x[\cos(\theta)\cos(\phi+\chi) - i\sin(\theta)\sin(\phi+\chi)] + \mathbf{e}_y[\cos(\theta)\sin(\phi+\chi) + i\sin(\theta)\cos(\phi+\chi)]$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are unit vectors, θ is the feed ellipticity, ϕ is the orientation of the ellipse $i = \sqrt{-1}$ and χ the parallactic angle given for an alt-az mounted antenna by: [1]

$$\chi = \tan^{-1}\left(\frac{\cos(lat)\sin(ha)}{\sin(lat)\cos(dec) - \cos(lat)\sin(dec)\cos(ha)}\right)$$

where lat is the antenna latitude, dec is the declination of the source and ha is the hour angle of the source.

The response of a given interferometer including the effects of phase calibration can be written as the following:

$$\begin{aligned} F_{jk}^{obs} = g_j g_k^* \{ & RR_{jk}[(\cos\theta_j + \sin\theta_j)e^{-i(\phi_j+\chi_j)}] \times [(\cos\theta_k + \sin\theta_k)e^{i(\phi_k+\chi_k)}] \\ & + RL_{jk}[(\cos\theta_j + \sin\theta_j)e^{-i(\phi_j+\chi_j)}] \times [(\cos\theta_k - \sin\theta_k)e^{-i(\phi_k+\chi_k)}] \\ & + LR_{jk}[(\cos\theta_j - \sin\theta_j)e^{i(\phi_j+\chi_j)}] \times [(\cos\theta_k + \sin\theta_k)e^{i(\phi_k+\chi_k)}] \\ & + LL_{jk}[(\cos\theta_j - \sin\theta_j)e^{i(\phi_j+\chi_j)}] \times [(\cos\theta_k - \sin\theta_k)e^{-i(\phi_k+\chi_2)}]\} \end{aligned}$$

where the effects of phase calibration are given by $g_R = e^{-i(-\chi-\phi_R+\phi_{Rref})}$ and $g_L = e^{i(-\chi-\phi_L+\phi_{Lref}+\phi_{R-L})}$, ϕ_{R-L} = Right - Left phase difference, RR_{jk}, LL_{jk}, RL_{jk} and LR_{jk} are the responses of interferometers with perfect right and left circular feeds to the source polarization. A least squares fit of the parameters of this model to data will require the partial derivatives of the above relation. These partial derivatives are given in Appendix A.

To include the effects of source resolution the values of RR_{jk}, LL_{jk}, RL_{jk} and LR_{jk} can be determined from the observed total intensity, $0.5(RR_{jk} + LL_{jk})$, and fractional Q, U and V values.

The determined corrections can be applied by making use of the matrix relationship between the polarization vectors:

$$F_{jk}^{obs} = M_{jk} F_{jk}^{true}$$

where F^{obs} are the observed stokes correlations and the F^{true} are the true values. This relationship can be inverted to determine the corrected Stokes' correlation vector:

$$F_{jk}^{corr} = M_{jk}^{-1} F_{jk}^{obs}$$

4.2 Linearized Model

For nearly perfect feeds and weakly polarized calibrators it is possible to model the response of the feeds with a "leakage" term:

$$R_k = G_{kR} e^{i(\chi_k)} (E_R e^{-i\chi_k} + D_{kR} E_L e^{i\chi_k})$$
$$L_k = G_{kL} e^{-i(\chi_k)} (E_L e^{i\chi_k} + D_{kL} E_R e^{-i\chi_k})$$

[1] Actual computation of the parallactic angle should involve a two argument arctangent function to resolve the quadrant ambiguities.

where E_R and E_L are the electric field strengths of the right and left hand polarizations and G_{kR} and G_{kL} are the complex gains needed to calibrate the amplitude and phase of the right and left circularly polarized feeds.

In the case of nearly perfect feeds and weakly polarized sources, second order terms in D and terms involving the product of D and polarized emission can be ignored leaving a linear expression in D and source polarization. If the approximations of no circular polarizations and similar total intensity and polarized structure is made then the response will be approximated by:

$$\frac{RL_{jk}^{obs}}{II} = \frac{(Q+iU)}{II} + D_{jR}\, e^{-2i\chi_j} + D_{kL}^{*}\, e^{-2i\chi_k}$$

$$\left(\frac{LR_{jk}^{obs}}{II}\right)^{*} = \frac{(Q+iU)}{II^{*}} + D_{kR}\, e^{-2i\chi_k} + D_{jL}^{*}\, e^{-2\chi_j}$$

where $II = 0.5(RR_{jk} + LL_{jk})$. A least squares fit of the parameters of this model to data will require the partial derivatives of this relation. These partial derivatives are given in Appendix B.

Using this model and data which has been amplitude and phase calibrated as described above, the two complex parameters D_R and D_L needed to describe each feed pair can be determined. To first order the observed values can then be corrected:

$$RL_{jk}^{corr} = RL_{jk}^{obs} - RR_{jk}^{obs} D_{kL}^{*} e^{-2i\chi_k} - LL_{jk}^{obs} D_{jR} e^{-2i\chi_j}$$

$$LR_{jk}^{corr} = LR_{jk}^{obs} - RR_{jk}^{obs} D_{jL} e^{2i\chi_j} - LL_{jk}^{obs} D_{kR}^{*} e^{2i\chi_k}$$

Since corrections to RR_{jk} and LL_{jk} depend only on the higher order terms ignored in the determination of the D terms they cannot be corrected using this approximation. This model can be used to correct RR and LL and improve its usefulness for feeds which are significantly non circular by including the higher order terms in source and antenna polarization when solving for the model. Unfortunately this causes the solutions to be nonlinear.

5 Ionospheric Faraday Rotation

During times of enhanced solar activity, Faraday rotation in the ionosphere may be significant, especially at lower frequencies (see Cotton 1989). Faraday rotation is a rotation of the apparent orientation of the linear polarization and will therefore change the right-left phase difference.

The exact amount of Faraday rotation depends on the integral of the product of the electron density and the component of the magnetic field which is parallel to the line of sight. This will cause strong variations of the Faraday rotation with the observing geometry. The electron density in the ionosphere has strong diurnal variations due to the variable exposure to the ionizing radiation (both photon and charged particle) from the sun.

The effects of Faraday rotation will therefore differ among sources observed at the same time but with different celestial positions and will vary with time for a given source. This will cause a variable right-left phase difference. Since this phase difference is not constant it can result in bogus estimates of the polarization parameters and can severely defocus the polarized image.

The amount of Faraday rotation can be estimated from a model of the ionosphere and the earth's magnetic field. The magnetic field is only weakly variable (in human experience but geological evidence says otherwise) but the electron density is strongly variable. The electron density may be estimated from a model based on solar activity as parameterized by mean sunspot number (Chiu 1975) of by direct measurement by one of a number of methods.

There are several possible techniques for correcting for Faraday rotation. Since the effects of Faraday rotation are similar to that of parallactic angle rotation, a correction for Faraday rotation can be made in the determination of polarization parameters and the application of calibration of the data by subtracting the Faraday rotation from the parallactic angle and using this value as the parallactic angle. If this technique is used then the Faraday rotation corrections must be included every time the parallactic angle is used including the initial correction of the phases for the effects of parallactic angle.

In principle, Faraday rotation could be determined from observations of a strongly polarized, unresolved source but in practice estimates of Faraday rotation are usually derived from external data. This being the case, a simpler method of removing Faraday rotation is to compute the effects once and adjust the relative phases of the right and left hand systems before determining or applying any corrections determined directly from the observations. To the degree that the estimates of Faraday rotation are accurate this should completely correct this effect.

6 Calibration of the Orientation of the E-vectors

The right and left handed systems of phases will be given an arbitrary offset by the effects of the earth's atmosphere and the electronics of the interferometer. With proper calibration this phase difference can be made the same for all data. This phase difference will still contain a component due to atmospheric and instrumental effects. The proper value for the right-left phase difference can only be determined from observations of a source with known polarization properties. In the following discussion it will be assumed that the polarization angle of the linear polarization integrated over the entire source is known for one or more calibrator sources.

The orientation of the polarization, i.e. the E-vectors of the electric field of the radiation, is by definition given by:

$$\Phi = \frac{1}{2}\tan^{-1}\frac{Q}{U}.$$

Since $RL = q + iu$ it follows that for a point source at the phase center the true right-left phase difference after removing atmospheric and instrumental effects is:

$$\phi_{R-L}^{true} = 2\Phi.$$

6.1 Unresolved Polarization Calibrator

If observations of a point source of known polarization orientation were made, the correction to the right-left phase difference can be determined by direct inspection of the calibrated RL (or LR^*) phases. However, it may be desirable to incorporate the right-left phase calibration into the normal phase calibration process. Unfortunately, phase calibration must be applied before the application of polarization corrections. This means that the instrumental polarization parameters must be suitably modified for the right-left phase correction.

An alternative method of applying the right-left phase difference is by suitable rotation of the polarization angles derived from the Q and U images. If the right-left phase difference is constant this will result in a constant rotation of the apparent polarization angle. Thus, the images in linear polarization will be correct except for a constant rotation of the orientation of the polarization angle.

6.2 Resolved Polarization Calibrator

It is frequently the case with VLBI observations that all calibrators are resolved. The observed RL and LR phases of a resolved source (or one not at the phase center) will not have a simple relation to the right-left phase difference and is therefore not directly usable for calibration. If the calibrator is unresolved on a subset of the baselines and then the right-left phase difference may be determined from these baselines.

If the calibrator is too resolved, the data are too noisy or the residual calibration errors are too high on the baselines for which the source is unresolved then the correction to the right-left phase difference must be determined from the derived image of the calibrator. This can be done using the integrated Q and U flux density in the images (e.g. sum of the CLEAN components). The apparent, integrated polarization angle is given by:

$$\Phi = \frac{1}{2}\tan^{-1}\frac{\Sigma Q}{\Sigma U}.$$

A correction to the apparent polarization angle or right-left phase difference can then be determined and then applied as in the unresolved calibrator case.

7 Imaging Considerations

The usual imaging technique for linear polarization is to form q and u values from the observed RL and LR correlations and separately image and deconvolve the Q and U images. This inhibits the use of data for which only one of RL or LR are available. There are many reasons why only one of the cross polarized correlations may be available but the impact may be fairly serious for observations using VLBI techniques as the uv plane sampling is usually rather sparse. If sufficient data exist with both cross polarized correlations then the traditional approach is adequate.

Single cross polarized measurements may be used to form an image of the linear polarization if a complex imaging and deconvolution of $Q + iU$ is done. In the aperture domain RL and LR sample conjugate uv coordinates. Since the $Q+iU$ image is complex its Fourier transform is in general assymetric so RL and LR measure different aspects of the source. Conversely, if the sampling function is assymetric (some measurements don't have both RL and LR) then the Fourier transform of the sampling function, the dirty beam, is complex. It is therefore possible to use assymetric sampling and produce a $Q + iU$ complex image and complex beam. A complex deconvolution should result in a complex image for which the real part represents the Q emission from the sky and the imaginary part the U emission. The CLEAN deconvolution algorithm is especially easily adapted as all operations used have complex analogs.

8 Suggested Method of Calibration

This section suggests a method of calibration and describes how this is accomplished using AIPS tasks in the 15APR92 or later releases. In the examples of instructions to AIPS comments are given in square brackets ([ ]).

If the polarization of the calibrator sources is unknown then the observations should include measurements of one or more calibrator sources over a range of parallactic angles. The observations of these calibrators will be used to determine the instrumental polarization parameters and should be calibrated in as nearly as possible the same manner as the program sources. The task LISTR can be used to examine the parallactic angles observed using:
>task='LISTR'; opty='GAIN'; inext='CL'; inver=0; dparm=9,0; go

1. Evaluation of phase cal.

   Multi IF VLBI data such as is obtained from the MkIII or VLBA systems normally use a tone injected into the feed (or later in the system) to measure the instrumental phase of the different parts of the bandpass (IFs in AIPS). These phase cal signals may or may not be present for any antenna and/or IF and may have trouble. AIPS task MK3IN, if it determines that the phase cal is coherent leaves these values as the phase in the CL table. If MK3IN determines that there was no phase cal present it blanks the CL table entries. In this latter case it is necessary to manually set the phase cals. A listing of the phase cal phases can be obtained using task LISTR by:

   >task='LISTR'; opty='GAIN'; inext='CL'; inver=0; dparm=1,0; go

   A graphical display can be obtained using task SNPLT:

   >task='SNPLT'; inext='CL', inver 0; bif=1; eif=0; opty='PHAS';

   > opcode='PLIF'; go

   If some of the phase cals are missing or badly behaved they should be set to zero using CLCOR and selecting the affected data:

   >task='CLCOR'; opcode='PCAL'; antenna=[list of antenna numbers];

   > bif=[?]; eif=[?]; gainver=1; CLCORPRM=0; go

   If valid phase cal phases are available they should be used as they should remove time variations in the instrumental phase. This may be especially critical for maintaining a constant phase relationship between the right and left handed systems. If possible the reference antenna should have good phase cal values. The phase cals set to zero will be set to appropriate values in a later step.

2. Check/correct antenna mount type.

   The mount type of each antenna determines the parallactic angle by which the source polarization is rotated. The mount type is stored in the antenna (AN) table. Usually the values are the default values corresponding to alt-az mounts. The values in the AN table can be examined using PRTAN:

   >task='PRTAN'; inver 1; ncount=0; go

   If any of the mount types need to be changed this can be done using verb TABPUT, first find the mount type column number and the row numbers of the antennas to be changed:

   >task='PRTAB'; inext='AN'; inver=1; ncount=0; go

   Note the column number for the column labeled "MNTSTA" and the row numbers of the antennas with incorrect mount types. A table entry can then be changed as follows (assuming the new mount is equatorial):

   >pixxy= [row no., column no., 1]; keyv=1,0; tabput

   It's a good idea to rerun PRTAN to be sure you've got it right.

3. A priori amplitude calibration.

   All data should have amplitude corrections applied using the usual techniques employing system temperature, antenna gain measurements, and calibrator source observations. Amplitude calibration is done using AIPS task ANCAL. Since the procedures for amplitude calibration differ little from those for single polarization VLBI data the reader is referred to the EXPLAIN documentation for ANCAL.

4. Correction for parallactic angle.

   The phases of all data should be corrected for the effects of parallactic angles. If a copy of the CL table was not done in the amplitude calibration step it should be done here in case the calibration needs to be restarted (quite likely). This is done using task TACOP:

   >task='TACOP'; clro; inext='CL'; inver=1; outver=2; ncount=1; go

   The correction for parallactic angle is done using task CLCOR.

   >task='CLCOR'; opcode='PANG'; gainver=2; ante=0; bif=1; eif=0;

   >stokes=' '; clcorprm=1; go

5. Correction for Faraday rotation.

   If ionospheric Faraday rotation is a problem then a correction to the phases of the data should be done at this point. If this correction is done by adjusting feed based gains then the phases of one set of feeds (e.g. all left circular) can be adjusted to remove the effects of Faraday rotation. This correction is done using task FARAD and the reader is referred to it's documentation. Note: for VLBI arrays FARAD may have to be run multiple times.

6. Right-left multi- and single-band delay difference correction.

   The cross polarized data from a short segment of data on a single baseline involving the reference antenna should be used to determine the delay differences between the right and left hand systems. If a multi-frequency system such as MkIII or VLBA recorders have been used then a phase difference in each frequency band should also be determined after applying any phase cal measurements. The application of corrections for these delay and phase measurements should align the single- and multi-band delays between the right and left handed systems.

   In 15OCT92 and later releases of AIPS this calibration step can be accomplished using the procedure CROSSPOL. See EXPLAIN CROSSPOL for details of its use. CROSSPOL can use data from several calibrators but the number of baselines is restricted. In older versions of AIPS the recommended procedure is to use SWPOL to switch the right and left circular data for one antenna, preferably the reference antenna, for a short segment of data on a strong calibrator. First use UVCOP to select a short section of calibrator data:

   >task='UVCOP'; timerang=[set time range]; antenna=[antennas];

   > basel=ante; outn=inn; outc='SHORT'; outs 0; go

   Since parallel hand delay and rate calibration has not already been done for this dataset, FRING should be run for the two antennas. The solution interval depends on the frequency, source strength, etc. but usually a few minutes is sufficient. The details of the values for DPARM may be different; see EXPLAIN FRING for details.

   >task='FRING'; inc='UVCOP'; snver=0; antenna=[refant, other]

   > docal=1; gainuse=0; refant=[ref. ant]; solint=[?];

   > aparm=2,0; dparm=1,500,50,2; go

   After FRING completes CLCAL can apply the solution to the prior CL table and produce a new CL table. Task SWPOL can then switch the polarization for the reference antenna:

   >task='SWPOL'; inc='SHORT'; ins 0; outc='SWPOL';

   >antenna=[ref. ant, 0]; docalib=1; go

   This will produce a data set in which all baselines to the reference antenna have their RR and LL data exchanged with the RL and LR data. Whether the "RR" data is actually RL or LR depends on whether the reference antenna has a higher or lower number that the other antenna making the baseline. If the reference antenna has a lower number then "RR" is actually LR and "LL" is actually RL; if the reference antenna number is higher then "RR" is RL and "LL" is LR.

The single band delay differences can be determined using FRING. FRING should apply the CL table produced in the previous CLCAL step and modified by SWPOL. Run FRING on the output of SWPOL as before:

```
>task='FRING'; inc='SWPOL'; inver=0; antenna=[refant, other];
> docal=1; gainuse=0; refant=[ref. ant]; solint=[?];
> aparm=2,0; dparm=1,500,50,2; go
```

The fitted values of the single band delays can be read from the SN table produced by FRING using LISTR

```
>task='LISTR'; opty='gain'; inext='CL'; inver 0; dparm 6 0; go
```

The value reported for "R" for the "other" antenna should have the opposite sign from that for "L". After possibly averaging in time and polarization (with the appropriate sign flip) these corrections can be entered into the CL table of your original data using CLCOR:

```
> tget CLCOR; opcode='SBDL'; clcorp=(list of IF single-band delays);
> stokes='L'; go
```

Running SHOUV can help determine if you got the signs correct.

```
>task='SHOUV'; opty='SPEC'; docal=1;gainuse=0;
> antenna=[ref. ant,other]; dparm=1,0,0,1[=averaging time];
> stokes='RL'; go
```

If all worked as it should the phases in each IF should be flat with frequency although there may be IF to IF differences.

The multi-band delay differences and any IF peculiar phase differences can be determined by averaging the data in frequency in each IF using SHOUV. Since 15APR92 and earlier versions of CLCOR will modify the multi-band delay when correcting the single band delay the following step should be done using the same data and calibration as was used in the last SHOUV step.

```
>task='SHOUV'; opty='AVIF'; stokes='RL' [or 'LR']; go
```

The phases should have opposite sign in RL and LR. The IF values should be referenced to the first IF by subtracting the phase in the first IF; this allows time averaging in the presence of nonzero fringe rate.

Suitable time/polarization averaged values can be used to correct the data using CLCOR:

```
>tget CLCOR; opco='POLR'; stokes=' ';
> clcorp=[list of IF phase corrections]; go
```

Run SHOUV again as before to be sure that the phases are constant in IF.

7. Fringe fit on calibrator to align delays and phases.

   For multi-frequency systems involving separate electronics for different parts of the measured cross-correlation spectrum it is usually necessary to align the phase and delays of various parts of the system. Run time phase calibration measurements may be available but currently existing systems are accurate to only 10 or 20 degrees and corrections may be needed. These corrections can be made using a short time segment on a strong calibrator source involving all antennas. A fringe fit to determine phase and delay for each separate set of electronics (video/baseband converter) is made to the short segment of calibrator data and then applied to all data. If fringe rates are determined in this process then the fringe rate corrections should be set to zero before application.

   The fringe fit is done using FRING. Some of the details may need to be different from the example; see the documentation for FRING. The calibrator source and timerange should specify a couple of minutes on a strong source during which all antennas/ IF etc were working.

   >task='FRING'; getn [original data]; calsour=[source]; docal=1; gainuse=0;

   > timerang=[appropriate timerange]; solint=15;

   > smodel=1,0; refant=[ref. ant.]; aparm=3,0,0,0,0,1;

   > dparm=1,1000,50,2; snver=1; go

   Examine FRING output or the SN table with LISTR to be sure that FRING worked OK. The quoted SNR for all antennas and IF should be acceptable (at least several 10s). There should be only one solution per antenna in the SN table.

   The fringe rates can be zeroed using SNCOR

   >task='SNCOR'; snver=1; opcode='ZRAT'; go

   This solution can be applied to all data using CLCAL

   >task='CLCAL'; gainver=2; gainuse=3; snver=1; opcode='CALI';

   >interp="; smotype="; refant=[ref. ant.]; go

8. Fringe fit.

   All data should then be fringe fitted solving for single-band plus multi-band delays if appropriate. The details may differ from the example shown below; consult the documentation for FRING to help determine the correct values of the parameters.

   Noise and differences in the time sampling of the left and right handed systems may result in different estimates of the residual fringe rates thus causing a decorrelation of the right and left hand systems. It is therefore necessary to average all rates determined at a single time for a given antenna before applying corrections. If the rates actually are different then the right and left hand systems are intrinsically incoherent and the data cannot be used to image polarized emission.

All phase like measurements (phase, delay and rate) should be referred to a common reference antenna if multiple phase reference antennas were used in the fringe fitting. Measured phase (etc.) differences between the primary and secondary reference antennas can be used to interpolate/extrapolate corrections to data referred to secondary antennas. This step is necessary to assure that a single set of right-left phase and delay differences are adequate for all data.

>task='FRING'; getn [original data]; calsour=' ';

> docal=1; gainuse=0; timerang=0; solint=4;

> smodel=1,0; refant=[ref. ant.]; aparm=3,0,0,0,2,1;

> dparm=1,1000,50,2,1; snver=2; go

Examine FRING output or the SN table with LISTR to be sure that FRING worked OK. The fitted delays and rates should be relatively consistent with time for a given antenna and most solutions should be present.

If the data set had to be divided up by time (e.g. for reasons of disk space) then the solution tables produced by FRING must be combined using TABED before the following step. In 15OCT92 and later use SNSMO to remove solutions outside of a specified range of values, average fringe rates for a given antenna/time, re-reference the solution to a common antenna, smooth and interpolate failed solutions. The details may vary from the example given below.

>task='SNSMO'; snver=2; antenna=0; timer=0; interpol='BOX';

>smotype ='VLMB'; refant=[ref. ant.]; bparm=0.25,0.01, 0.25, 0.25,0.01;

>cparm=0.25,0.25,0.25,0.25,0, 0, 10, 50, 30; go

In the 15APR92 release this step requires using 1) SNCOR with opcode='AVRT' to average fringe rates, 2) CLCAL with opcode='SMOO' but interpol='' , intparm=0; smotype='' to re-reference the solutions to a common antenna. 3) SNCOR will remove wild solutions if necessary (see explain SNCOR). 4) SNSMO with SMOTYPE='VLMB' to smooth/interpolate solutions. 5) Another run of SNCOR with opcode='AVRT' will insure that all rates for a given antenna/time are the same.

At this point it is prudent to examine the fringe fit phase solutions to check for coherence between the right and left hand systems. The difference between the right and left hand phases determined for the same time should be constant. If not, a problem has occurred in the previous steps.

The consistency of the solutions can be checked using SNPLT. If a multi-band fit (APARM(5)>0) was done in FRING examining the phases in a single IF (use 1 prior to 15OCT92) is sufficient. If single band fits were done then all IFs must be examined independently.

>task='SNPLT'; inext='SN', inver 2; bif=1; eif=1; opty='PHAS';

>opcode=''; stokes='DIFF'; ncount=5; go

The most serious potential problem is the failure of solutions in one of the two polarizations of a primary or secondary reference antenna (a reference antenna used when the primary antenna is unavailable) in the first solution interval after a period of data during which one of them was not available. Prior to 15OCT92 this can cause problems in the re-referencing routines. Editing of the data and or solution table and re-running FRING and/or the SN table filtering routines may help restore R-L coherence.

These solutions can be applied using CLCAL

>task='CLCAL'; gainver=3; gainuse=4; snver=2; opcode='CALI';

>interp='SELF'; smotype=''; refant=[ref. ant.]; go

9. Phase calibration (self-calibration)

   The phase calibration of all data (calibrator and program sources) should be done using the usual techniques (self-calibration). Phase corrections determined from the parallel polarized data should be applied to the cross polarized data. In AIPS this is usually done with a combination of CALIB and MX. The procedure MAPIT (see EXPLAIN MAPIT) may simplify this process.

10. Determine instrumental polarization.

    The fully amplitude and phase calibrated observations of the calibrator source(s) should be used to determine the instrumental polarization parameters and the calibrator source polarization if unknown. This may involve iterating through the calibration/imaging process several times to converge on a polarization model for the calibrator(s). The model of the instrumental polarization depends on the quality of the feeds of the antennas used; if all are well behaved (or at least similar) then the linearized approximation will probably be adequate. For poor feeds the ellipticity-orientation model is required. In some cases the feed polarization parameters may be frequency dependant. The same reference antenna as was used for fringe fitting and phase calibration should be used for polarization calibration.

    The fully self-calibrated calibrator data set should be converted to a multi-source data set (containing one source) using task MULTI. The instrumental polarization determination is done using task PCAL. If the antenna feeds are poor (usually the case) the relatively expensive soltype='ORI-'

must be used for the ellipticity-orientation model. If the feeds are well behaved use the soltype='RAPR' linear approximation model.

>task='PCAL'; solint=4; calsour='[calibrator source]';

>soltype='ORI-'; prtlev=1; refant=[ref. ante.]; bparm=1,0,0,0,0,1;

>cparm=0; go

If the solutions are consistent in IF then PCAL can be run again with CPARM(1)=1 to average in IF before doing the solution. If the calibrator is unresolved and the polarization is known (or resolved and completely unpolarized) the polarization model can be given in adverb PMODEL and fitting to source polarization can be turned off using bparm(10)=1. If the calibrator is significantly resolved and the polarization structure differs from the total intensity (to be expected) then it is necessary to iterate through PCAL, imaging and deconvolution. The first time PCAL is run it should be allowed to fit for fractional Q and U. This solution should be applied to the calibrator as described below, and the resulting set of I, Q, and U CLEAN images given to the next run of PCAL as the source polarization model (IN2NAME, IN2CLASS, IN2SEQ, IN2DISK, NCOMP and NMAPS).

If multiple calibrator sources are to be used then the source polarization of each needs to be subtracted using UVSUB. The different sources should then be concatenated using MULTI and DBCON. PCAL using this subtracted dataset should be told not to solve for source polarization (BPARM(10)=1).

11. Correct for instrumental polarization.

    Properly amplitude and phase calibrated data should have the appropriate correction for the feed polarization applied. Task SPLIT can apply polarization corrections using the parameters in the AN table using DOPOL=true for either a single- or multi-source data set. The input to SPLIT for polarization calibration should be a fully self calibrated data set.

12. Image.

    Images can then be formed and deconvolved in Stokes' I, Q and U polarization. Task MX or HORUS and APCLN can be used to image and deconvolve the Stokes I, Q and U data. These routines can only use data for which both the RL and LR correlations are present in each visibility used.

    In the 15OCT92 and later releases assymetric polarization uv coverage can be used with a combination of the procedure CXPOLN and task CXCLN. CXPOLN uses UVPOL and MX to make complex dirty and beam images. CXCLN will then deconvolve these images and produce Q and U deconvolved images. See the documentation for CXPOLN and CXCLN. CXCLN produces a CX table with the complex CLEAN components attached to the deconvolved Q image. If a pair of CC tables containing the Q and U clean components is desired (e.g. for PCAL) then task TBSUB can make the conversion.

>task='TBSUB'; inex='CX'; inver=1; IN2EXT='CC'; outver=1; bcount=1;

> ecount=0; sourc='COL#(3)','COL#(1)','COL#(2)';

> calsour='FLUX','DELTAX','DELTAY' ; go

> SOURC(1)='COL#(4)'; outver=2;go

This will leave the Q components in CC table 1 and the U components in CC table 2 which can be copied to the U deconvolved image using task TACOP.

13. Correction of position angle of E-vectors. The proceeding process will leave the derived images with an arbitrary offset in the apparent orientation of the E-vector of the image. Correction requires knowledge of the orientation of the integrated polarized emission of at least one observed source (presumably a calibrator). The correction to the polarization angles can be determined from the integrated emission in the derived image and then applied to all sources.

    The calibration of polarization angle can be done either in COMB by adding an offset to the polarization angle when making the polarization angle image or to the uv data using CLCOR and opcode='POLR'. CLCOR must be used on a multi-source data set before running SPLIT. This correction may be applied before the self calibration and polarization calibration of the program source but it is rather inconvenient to go back to this stage for the calibrator source(s). The apparent integrated polarization angle is given by:

$$PA = \frac{1}{2} tan^{-1}(\frac{\Sigma U}{\Sigma Q})$$

    where ΣU is the sum of the U CLEAN components and ΣQ is the sum of the Q clean components. (Note: use a two argument arctangent function.) The correction to apply in CLCOR or COMB is the true polarization angle minus the apparent polarization angle. CLCOR can be run as follows.

    >task='CLCOR'; opcode='POLR'; gainver=4; ante=0; bif=1; eif=0;

    >stokes=' ';clcorprm=[PA correction]; go

9 Acknowledgments

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10 References

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Appendix A

Formulae for Ellipticity-Orientation Model

This appendix gives the partial derivatives of the Ellipticity-Orientation model given in Section 5.1 with respect to the model parameters. The model can be factorized into antenna based components using the following:

$$S_{Ri} = \cos\theta_{Ri} + \sin\theta_{Ri}$$

$$D_{Ri} = \cos\theta_{Ri} - \sin\theta_{Ri}$$

$$S_{Li} = \cos\theta_{Li} + \sin\theta_{Li}$$

$$D_{Li} = \cos\theta_{Li} - \sin\theta_{Li}$$

$$P_{Ri} = e^{2i\phi_{Ri}}$$

$$P_{Li} = e^{-2i\phi_{Li}}$$

The model values of the correlations RR_{jk}, RL_{jk}, LR_{jk}, and LL_{jk} can either be provided from the Fourier transform a model of the source emission or by using the similarity approximation and the following relations:

$$RR_{jk} = (I+V)\,0.5(RR^{obs}_{jk} + LL^{obs}_{jk})$$

$$RL_{jk} = (Q+iU)\,0.5(RR^{obs}_{jk} + LL^{obs}_{jk})$$

$$LR_{jk} = (Q-iU)\,0.5(RR^{obs}_{jk} + LL^{obs}_{jk})$$

$$LL_{jk} = (I-V)\,0.5(RR^{obs}_{jk} + LL^{obs}_{jk}).$$

Here I is the fractional total intensity (=1). Q, U, and V are the fractional polarizations.

A Partial Derivatives for RR Correlations

Compute the components of the model:

$$C_{RR} = 2S_{Rj}S_{Rk}$$

$$C_{RL} = 2S_{Rj}D_{Rk}P^*_{Rk}e^{2i\chi_k}$$

$$C_{LR} = 2D_{Rj}P_{Rj}S_{Rk}e^{-2i\chi_j}$$

$$C_{LL} = 2D_{Rj}P_{Rj}D_{Rk}P^*_{Rk}e^{-2i(\chi_j-\chi_k)}$$

$$RR^{mod}_{jk} = RR_{jk}C_{RR} + RL_{jk}C_{RL} + LR_{jk}C_{LR} + LL_{jk}C_{LL}$$

$$\frac{\partial RR}{\partial \phi_{Rj}} = 4iD_{Rj}e^{-2i\chi_j}(LL_{jk}D_{Rk}P^*_{Rk}e^{2i\chi_k} + LR_{jk}S_{Rk})$$

$$\frac{\partial RR}{\partial \phi_{Rk}} = -4iD_{Rk}e^{2i\chi_k}(LL_{jk}D_{Rj}P_{Rj}e^{-2i\chi_j} + RL_{jk}S_{Rj})$$

$$\frac{\partial RR}{\partial \theta_{Rj}} = 2\{D_{Rj}(RR_{jk}S_{Rk} + RL_{jk}D_{Rk}P^*_{Rk}e^{2i\chi_k})$$

$$-S_{Rj}P_{Rj}e^{-2i\chi_j}(LR_{jk}S_{Rk} + LL_{jk}D_{Rk}P^*{}_{Rk}e^{2i\chi_k})\}$$

$$\frac{\partial RR}{\partial \theta_{Rk}} = 2\{D_{Rk}(RR_{jk}S_{Rj} + LR_{jk}D_{Rj}P_{Rj}e^{-2i\chi_j})$$

$$-S_{Rk}P^*_{Rk}e^{2i\chi_k}(RL_{jk}S_{Rj} + LL_{jk}D_{Rj}P_{Rj}e^{-2i\chi_j})\}$$

$$\frac{\partial RR}{\partial I} = 0.5[RR_{jk} + LL_{jk}](C_{RR} + C_{LL})$$

$$\frac{\partial RR}{\partial Q} = 0.5[RR_{jk} + LL_{jk}](C_{RL} + C_{LR})$$

$$\frac{\partial RR}{\partial U} = i0.5[RR_{jk} + LL_{jk}](C_{RL} - C_{LR})$$

$$\frac{\partial RR}{\partial V} = 0.5[RR_{jk} + LL_{jk}](C_{RR} - C_{LL})$$

B Partial Derivatives for RL Correlations

Compute the components of the model:

$$PP^* = e^{i(\phi_{Rref} + \phi_{Lref} + \phi_{R-L})}$$

$$C_{RR} = 2S_{Rj}S_{Lk}P^*_{Lk}e^{-2i\chi_k}$$

$$C_{RL} = 2S_{Rj}D_{Lk}$$

$$C_{LR} = 2D_{Rj}P_{Rj}S_{Lk}P^*_{Lk}e^{-2i(\chi_j+\chi_k)}$$

$$C_{LL} = 2D_{Rj}P_{Rj}D_{Lk}e^{-2i\chi_j}$$

$$RL^{mod}_{jk} = PP^*(RR_{jk}C_{RR} + RL_{jk}C_{RL} + LR_{jk}C_{LR} + LL_{jk}C_{LL})$$

If j is the reference antenna then

$$\frac{\partial RL}{\partial \phi_{Rj}} = 2ie^{i(\phi_{Lref} + \phi_{R-L})}D_{Rj}e^{-i(\phi_{Rref} + 2\chi_j)}(LL_{jk}D_{Lk} + LR_{jk}S_{Lk}P^*_{Lk}e^{-2i\chi_k})$$

else

$$\frac{\partial RL}{\partial \phi_{Rj}} = 4iPP^*e^{-2i\chi_j}D_{Rj}(LL_{jk}D_{Lk} + LR_{jk}S_{Lk}P^*_{Lk}e^{-2i\chi_k})$$

and

$$\frac{\partial RL}{\partial \phi_{Rref}} = iRL_{jk}^{mod}$$

If k is the reference antenna then

$$\frac{\partial RL}{\partial \phi_{Lk}} = 2ie^{i(\phi_{Rref}+\phi_{R-L})}S_{Lk}e^{-i(\phi_{Lref}+2\chi_k)}(RR_{jk}S_{Rj} + LR_{jk}D_{Rj}P_{Rj}e^{-2i\chi_j})$$

else

$$\frac{\partial RL}{\partial \phi_{Lk}} = 4iPP^* e^{-2i\chi_k}S_{Lk}(RR_{jk}S_{Rj} + LR_{jk}D_{Rj}P_{Rj}e^{-2i\chi_j})$$

and

$$\frac{\partial RL}{\partial \phi_{Lref}} = iRL_{jk}^{mod}$$

$$\frac{\partial RL}{\partial \theta_{Rj}} = 2PP^*\{D_{Rj}(RR_{jk}S_{Lk}P_{Lk}^* e^{-2i\chi_k} + RL_{jk}D_{Lk})$$

$$-S_{Rj}P_{Rj}e^{-2i\chi_j}(LR_{jk}S_{Lk}P_{Lk}^* e^{-2i\chi_k} + LL_{jk}D_{Lk})\}$$

$$\frac{\partial RL}{\partial \theta_{Lk}} = 2PP^*\{D_{Lk}P_{Lk}^* e^{-2i\chi_k}(RR_{jk}S_{Rj} + LR_{jk}D_{Rj}P_{Rj}e^{-2i\chi_j})$$

$$-S_{Lk}(RL_{jk}S_{Rj} + LL_{jk}D_{Rj}e^{-2i\chi_j})\}$$

$$\frac{\partial RL}{\partial I} = 0.5[RR_{jk} + LL_{jk}](C_{RR} + C_{LL})$$

$$\frac{\partial RL}{\partial Q} = 0.5[RR_{jk} + LL_{jk}](C_{RL} + C_{LR})$$

$$\frac{\partial RL}{\partial U} = i0.5[RR_{jk} + LL_{jk}](C_{RL} - C_{LR})$$

$$\frac{\partial RL}{\partial V} = 0.5[RR_{jk} + LL_{jk}](C_{RR} - C_{LL})$$

$$\frac{\partial RL}{\partial \phi_{R-L}} = iRL_{jk}^{mod}$$

C Partial Derivatives for LR Correlations

Compute the components of the model:

$$PP^* = e^{-i(\phi_{Lref}+\phi_{Rref}+\phi_{R-L})}$$

$$C_{RR} = 2S_{Lj}P_{Lj}S_{Rk}e^{2i\chi_j}$$

$$C_{RL} = 2S_{Lj}P_{Lj}D_{Rk}{P^*}_{Rk}e^{2(i\chi_j+\chi_k)}$$

$$C_{LR} = 2D_{Lj}S_{Rk}$$

$$C_{LL} = 2D_{Lj}D_{Rk}P^*_{Rk}e^{2i\chi_k}$$

$$LR^{mod}_{jk} = PP^*(RR_{jk}C_{RR} + RL_{jk}C_{RL} + LR_{jk}C_{LR} + LL_{jk}C_{LL})$$

If j is the reference antenna then

$$\frac{\partial LR}{\partial \phi_{Lj}} = -2ie^{-i(\phi_{Lref}+\phi_{R-L})}D_{Rk}e^{i(\phi_{Rref}+2\chi_k)}(LL_{jk}D_{Lj} + RL_{jk}S_{Lj}P_{Lj}e^{2i\chi_j})$$

else

$$\frac{\partial LR}{\partial \phi_{Lj}} = -4iPP^*e^{2i\chi_j}S_{Lj}(RR_{jk}S_{Rk} + RL_{jk}D_{Rk}P^*_{Rk}e^{2i\chi_k})$$

and

$$\frac{\partial LR}{\partial \phi_{Lref}} = -iLR^{mod}_{jk}$$

If k is the reference antenna then

$$\frac{\partial LR}{\partial \phi_{Rk}} = -2ie^{i(\phi_{Rref}-\phi_{R-L})}S_{Lj}e^{-i(\phi_{Rref}-2\chi_j)}(RR_{jk}S_{Rk} + RL_{jk}D_{Rk}P^*_{Rk}e^{2i\chi_k})$$

else

$$\frac{\partial LR}{\partial \phi_{Rk}} = -4iPP^*e^{2i\chi_k}D_{Rk}(LL_{jk}D_{Lj} + RL_{jk}S_{Lj}P_{Lj}e^{2i\chi_k})$$

and

$$\frac{\partial LR}{\partial \phi_{Rref}} = -iLR^{mod}_{jk}$$

$$\frac{\partial LR}{\partial \theta_{Lj}} = 2PP^*\{D_{Lj}P_{Lj}e^{2i\chi_j}(RR_{jk}S_{Rk} + RL_{jk}D_{Rk}P^*_{Rk}e^{2i\chi_k})$$

$$-S_{Lj}(LR_{jk}S_{Rk} + LL_{jk}D_{Rk}P^*_{Rk}e^{2i\chi_k})\}$$

$$\frac{\partial LR}{\partial \theta_{Rk}} = 2PP^*\{D_{Rk}(RR_{jk}S_{Lj}P_{Lj}e^{2i\chi_j} + LR_{jk}D_{Lj})$$

$$-S_{Rk}P^*_{Rk}e^{2i\chi_k}(RL_{jk}S_{Lj}P_{Lj}e^{2i\chi_j} + LL_{jk}D_{Lj})\}$$

$$\frac{\partial LR}{\partial I} = 0.5[RR_{jk} + LL_{jk}](C_{RR} + C_{LL})$$

$$\frac{\partial LR}{\partial Q} = 0.5[RR_{jk} + LL_{jk}](C_{RL} + C_{LR})$$

$$\frac{\partial LR}{\partial U} = i0.5[RR_{jk} + LL_{jk}](C_{RL} - C_{LR})$$

$$\frac{\partial LR}{\partial V} = 0.5[RR_{jk} + LL_{jk}](C_{RR} - C_{LL})$$

$$\frac{\partial RL}{\partial \phi_{R-L}} = -iLR^{mod}_{jk}$$

D Partial Derivatives for LL Correlations

Compute the components of the model:

$$C_{RR} = 2S_{Lj}P_{Lj}S_{Lk}P^*_{Lk}e^{2i(\chi_j-\chi_k)}$$

$$C_{RL} = 2S_{Lj}P_{Lj}D_{Lk}e^{2i\chi_j}$$

$$C_{LR} = 2D_{Lj}S_{Lk}P^*_{Lk}e^{-2i\chi_k}$$

$$C_{LL} = 2D_{Lj}D_{Lk}$$

$$LL^{mod}_{jk} = RR_{jk}C_{RR} + RL_{jk}C_{RL} + LR_{jk}C_{LR} + LL_{jk}C_{LL}$$

$$\frac{\partial LL}{\partial\phi_{Lj}} = -4iS_{Lj}e^{2i\chi_j}(RR_{jk}S_{Lk}P^*_{Lk}e^{-2i\chi_k} + RL_{jk}D_{Lk})$$

$$\frac{\partial LL}{\partial\phi_{Lk}} = 4iS_{Lk}e^{-2i\chi_k}(RR_{jk}S_{Lj}P_{Lj}e^{2i\chi_j} + LR_{jk}D_{Lj})$$

$$\frac{\partial LL}{\partial\theta_{Lj}} = 2\{D_{Lj}e^{2i\chi_j}P_{Lj}(RR_{jk}S_{Lk}P^*_{Lk}e^{-2i\chi_k} + RL_{jk}D_{Lk})$$
$$-S_{Lj}(LR_{jk}S_{Lk}P^*_{Lk}e^{-2i\chi_k} + LL_{jk}D_{Lk})\}$$

$$\frac{\partial LL}{\partial\theta_{Lk}} = 2\{D_{Lk}P^*_{Lk}e^{-2i\chi_k}(RR_{jk}S_{Lj}P_{Lj}e^{2i\chi_j} + LR_{jk}D_{Lj})$$
$$-S_{Lk}(RL_{jk}S_{Lj}P_{Lj}e^{2i\chi_j} + LL_{jk}D_{Lj})\}$$

$$\frac{\partial LL}{\partial I} = 0.5[RR_{jk} + LL_{jk}](C_{RR} + C_{LL})$$

$$\frac{\partial LL}{\partial Q} = 0.5[RR_{jk} + LL_{jk}](C_{RL} + C_{LR})$$

$$\frac{\partial LL}{\partial U} = i0.5[RR_{jk} + LL_{jk}](C_{RL} - C_{LR})$$

$$\frac{\partial LL}{\partial V} = 0.5[RR_{jk} + LL_{jk}](C_{RR} - C_{LL})$$

Appendix B

Formulae for Linearized Model

This appendix gives the partial derivatives for the linearized model given in section 5.2. The model values of the correlations RR_{jk}, RL_{jk}, LR_{jk}, and LL_{jk} can either be provided from the Fourier transform a model of the source emission or by using the similarity approximation and the following relations:

$$RR_{jk} = (I + V)\,0.5(RR_{jk}^{obs} + LL_{jk}^{obs})$$

$$RL_{jk} = (Q + iU)\,0.5(RR_{jk}^{obs} + LL_{jk}^{obs})$$

$$LR_{jk} = (Q - iU)\,0.5(RR_{jk}^{obs} + LL_{jk}^{obs})$$

$$LL_{jk} = (I - V)\,0.5(RR_{jk}^{obs} + LL_{jk}^{obs}).$$

Here I is the fractional total intensity (=1). Q, U, and V are the fractional polarizations.

The partial derivatives are given by the following.

$$\frac{\partial RL}{\partial D_{Rj}} = e^{-2i\chi_j}$$

$$\frac{\partial RL}{\partial D_{Lj}^*} = e^{-2i\chi_k}$$

$$\frac{\partial RL}{\partial Q + iU} = \frac{1}{0.5[RR + LL]}$$

$$\frac{\partial LR}{\partial D_{Lj}^*} = e^{-2i\chi_j}$$

$$\frac{\partial LR}{\partial D_{Rk}} = e^{-2i\chi_k}$$

$$\frac{\partial LR}{\partial Q + iU} = \frac{1}{0.5[RR + LL]^*}$$