$\mathcal{AIPS}$ Memo 122 Revised

Modeling Absorption-line Cubes in $\mathcal{AIPS}$

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Abstract

$\mathcal{AIPS}$ does Gaussian fitting of spectral lines with overhauled task XGAUS and can fit V polarization image cubes for Zeeman splitting with the task ZEMAN. Both of these tasks are designed for emission spectra in which the noise is not a function of spectral channel. In absorption, however, the noise in optical depth becomes high when the optical depth is high. Therefore, new tasks AGAUS and ZAMAN have been written to provide similar functions but with mathematics suitable for absorption lines. This memo describes the new tasks in some detail and includes a description of a new, simplified modeling task MODAB which may also be useful. That task has shown that the results of these four tasks are biased by the presence of the Zeeman splitting and need modest correction if they are meant to describe the actual pre-splitting line widths and magnetic field. [1]

1 Introduction

XGAUS was written a long time ago in the hopes that converting a spectral cube, with many spectral channels at each spatial location, into one or more sets of images of peak, center, width, and integral would simplify the data presentation. One could even hope that the separate Gaussians fit might have some separate physical reality. XGAUS was extensively re-written beginning in 2013 and is now a quite friendly and powerful task. Accompanying this task is ZEMAN which first appeared in 2013 and which solves for Zeeman splitting of the right and left polarizations using a new Gaussian model or the older methods involving the derivative of the unpolarized spectra. These tasks are described in detail in $\mathcal{AIPS}$ Memo 118 (revised)[2]

However, these tasks are designed for emission lines for which the noise is not a significant function of spectral channel. In absorption, the optical depth is the physical parameter which one might expect to be Gaussian. However, the noise in optical depth is a significant function of spectral channel whenever the optical depths become large. This calls for a new fitting routine to use Gaussian models in optical depth, but to fit the actual spectra for which the noise is approximately independent of spectral channel. This new task is named AGAUS and is very similar to XGAUS, but with enough differences that combining the two functions in one task is impractical.

A revision of ZEMAN, called ZAMAN, has also been written to solve for Zeeman splitting using the appropriate mathematics for absorption-line Gaussians. That mathematics will be presented below in some detail. A simple modeling task was written to prepare image cubes of known properties in order to test these new tasks. It has made it clear that the answers from XGAUS and AGAUS are affected by the Zeeman splitting as are the answers from ZEMAN and ZAMAN. The nature of the bias in the answers will be discussed below and correction methods suggested.

[1] Color is used to indicate changes made to this document: for the April, June and September, 2025 revisions.

[2] Greisen, Eric W., 2013 2025, "Modeling Spectral Cubes in $\mathcal{AIPS}$", AIPS Memo 118, NRAO, Socorro, NM.

2 Absorption-line Gaussian fitting: AGAUS

In absorption, we assume that the optical depth is proportional to the number of atoms/molecules in the line of sight and that the radial velocities of these absorbers is approximately Gaussian. Thus the function we seek to fit is

$$T(x, y, z) = (A(y, z) + B(y, z) * x) \times \exp(-\tau(x, y, z)) \quad (1)$$

where

$$\tau(x, y, z) = \sum_{1}^{\texttt{NGAUSS}} \tau_i e^{-\alpha(x-x_i)^2/\sigma_i^2} \quad (2)$$

where x is the coordinate along the first axis of the image cube, $A(y, z)+B(y, z)x$ is the spectral baseline (the continuum), $\tau_i(y, z)$ are the peak optical depths of each of the NGAUSS components, $x_i(y, z)$ are the center channels of the components, $\sigma_i(y, z)$ are the full widths at half maximum of the components and $\alpha = 4\ln(2)$ to impart this meaning to the σ_i.

AGAUS will fit this formulation to every row of a Stokes I image cube. Note that this cube must contain the full continuum emission of the source ($A(y, z)$) in order to be described by these equations. It then produces images over the second and third input axes of the fit parameters. Normal usage has the first axis of the cube as a spectral axis, either in frequency or velocity units, and the second and third axes are celestial coordinates. This usage will be assumed in the later discussion.

2.1 Inputs

The usual INNAME *et al.* adverbs define the cube to be fit and the OUTNAME *et al.* adverbs define the (eventual) output name. The INVERS adverb controls which XG table is used by the task, with ≤ 0 meaning a new table. BLC and TRC define the pixel ranges to be used in the current execution, where BLC(1) and TRC(1) control the spectral channels that will be fit and BLC(2), TRC(2), BLC(3), and TRC(3) control the area in celestial coordinates over which the fitting is done. New XG table files are created for the entire input image cube, but are filled with peak brightnesses limited by BLC(1) through TRC(1). Therefore, it would be wise to use as much of the first axis as is reliable when creating a new table. The second and third values of these adverbs then limit the area over which fitting is done during this execution of AGAUS. Adverbs YINC and ZINC control the stride taken in the first pass through the cube; a second pass will then fit all voxels not fit in the first pass. FLUX controls which spectra will be fit during this execution; all positions with 3 consecutive channels averaging above FLUX will be fit. The initial guess for the linear baseline (continuum brightness) is taken as the peak value with no slope. ORDER controls the order of the baseline with ≤ 0 meaning a constant and ≥ 1 meaning a constant plus a slope. DOOUTPUT controls what files are written — this may be changed interactively (except for one-dimensional images) so leave it zero at this point. Set DOTV = 1 to use the task interactively. Even when fitting only 1 Gaussian component, it is best to watch what is happening so you should never set this adverb false. DORESID controls whether residuals are plotted on the fit spectra. Such plots often provide clues when more Gaussians are needed for the best fits. However, the plots are self-scaled from the data only and so may not include the residuals which should be near zero. Set PIXRANGE if you must see the residuals. In general it is best to set PIXRANGE to zero to see the full range of image values in best detail, but, if you are fitting weak Gaussians in the presence of very strong ones, you might wish to cut off the lowest values. This will not interfere with setting initial guesses as the plot of apparent optical depth is always self-scaled. Note that, in 31DEC25, PIXRANGE may be set interactively so its value at this point is not important. Set LTYPE to your favorite type of labeling, set PIXVAL to zero to see all positions being fit, and leave NITER zero since 100 is more than enough iterations. Set NGAUSS to the number of Gaussians to be fit in this execution. If a new XG table is created, NGAUSS sets the maximum number of components that may appear in that table. In 31DEC25, NGAUSS may be as large as 32, while previous versions limited NGAUSS to 8 and always set the maximum in the table to eight. NGAUSS may be changed if you re-start on a pre-existing XG table but only with less than or equal that original number of Gaussians. A task to get around this limitation has also appeared in 31DEC25. XG2XG IS discussed in $\mathcal{AIPS}$ Memo 118. RMSLIMIT is an upper limit for the rms of a fit before the fit is viewed as "failed" which causes the TV and interaction to be turned back on after you have turned it off. You should get a good idea of an appropriate value from your initial uses of AGAUS or from your knowledge of the noise in your data cube.

Just as in XGAUS, AGAUS begins by creating an XG table and populating each row with the largest average brightness over three consecutive channels in the corresponding row in the range BLC(1) through TRC(1).. Then it reads the table every YINC rows and ZINC planes and, for those with an average brightness greater than FLUX, attempts a fit. Your interaction with this fit will be described below. After the first pass, the task loops over every row and plane fitting those positions which have enough brightness and which have not already been fit. Finally, after all voxels above FLUX have been fit, the task goes into an "edit" mode. It constructs images of each fit parameter and the integral flux of the Gaussian, and of the uncertainties in these parameters. In 31DEC25, it also constructs an image of the rms over the x axis for every (y, z) pixel. This image has a value at every pixel unlike the parameter images which have non-blanked values only at the pixels where the fit was regarded as good. You may view these images, select positions explicitly or by their parameter values or rms and revisit the fits of those positions. This stage will be described in detail below.

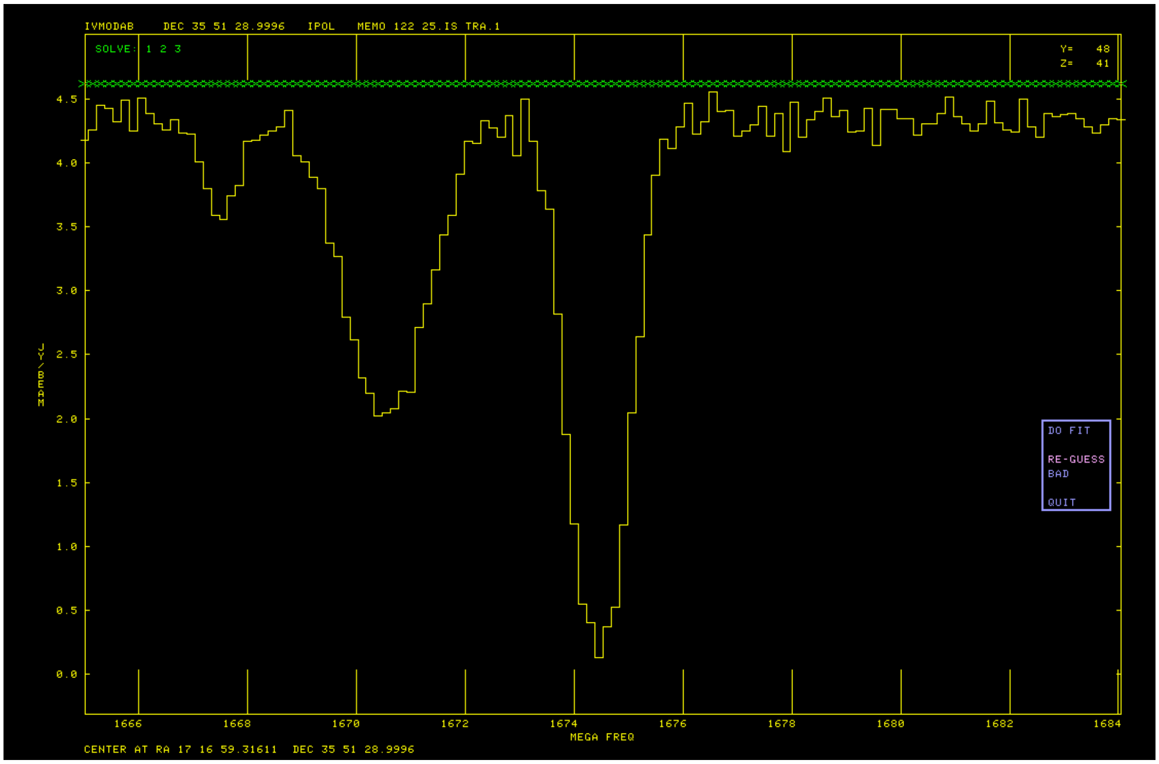
At any time you may exit the task and then re-start it using the same XG table. Good reasons for doing this include fitting smaller regions with each pass using the appropriate number of Gaussians for that region. Doing small regions which will have similar parameter values helps a great deal with the initial guessing done by the task (mostly using the previous solution). You might also fit the cube initially with a high value of FLUX and then re-start with a lower value to extend the areas fit.

2.2 Fitting

The fitting process starts with a plot of the spectrum with the data and axis labels in graphics channel one (usually yellow) and the initial guess as X's in graphics channel two (usually green). Then you are offered a menu of options on the TV (XAS window). We use a simplified source model in all figures for clarity. The model is the sum of two models computed by MODAB. The first spectrum to be fit is illustrated in Figure 1 at the top showing that the first guess for more than one Gaussian is not useful. The menu that appears at this point is

DO FIT	Attempt a fit
RE-GUESS	Interactively enter a new initial guess
BAD	Mark pixel bad and go on
QUIT	Exit AGAUS now

You select a menu option by moving the cursor to the desired option with the mouse and registering that move with the TV by clicking the left mouse button. The selected menu item will change color as shown in the figure. If you press TV "button" D at this point (actually keyboard character D), helpful information about the selected item will appear in your terminal window. If you press one of TV "buttons" A, B, or C (actually keyboard characters A, B, C), the selected function will be performed. The option to QUIT causes the task to quit at this point. You may re-start later. The option BAD will mark this position as failed and go on to the next position. The option DO FIT will cause the task to attempt the non-linear fit with the current initial guess. The selected option in Figure 1 is RE-GUESS which causes the task to change the plot to a spectrum of apparent optical depth, such as that shown in the figure at bottom. It will then prompt you first to *"Position cursor at center & height of Gaussian component 1"*. Move the cursor to the peak of component 1 and press any TV button. This selects the peak value and center of component 1. Then the task prompts you to *"Position cursor at half-width of Gaussian component 1"*. Move the cursor horizontally to the approximate position of the half-power point of component 1 and press any TV button. The horizontal position of the cursor then sets the initial guess of the full width of the component. These prompts are then repeated for components two through NGAUSS. If you do not want to fit a particular component at this position, move the cursor outside the rectangular border line (*i.e.*, outside the data area of the plot) before pressing the TV button for that component; it will be omitted.



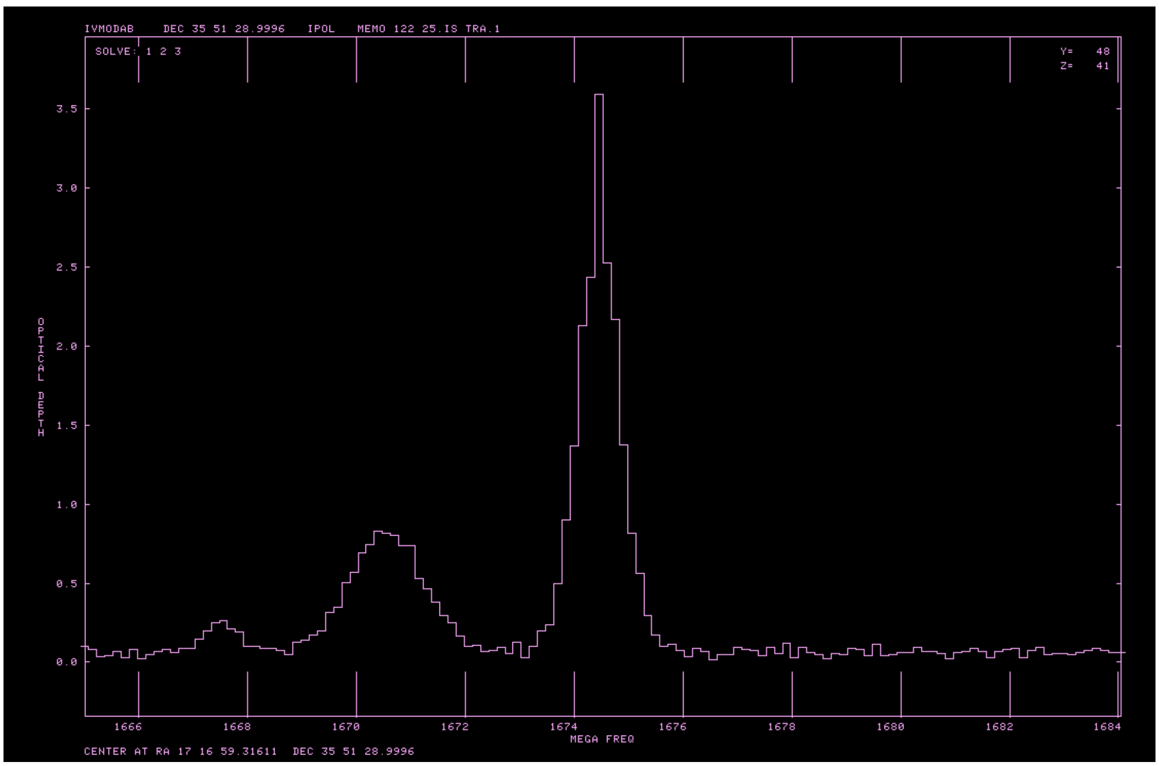


Figure 1: Top: First spectrum to fit, initial "guess" is all zero so `RE-GUESS` will be selected. Bottom: First spectrum to fit: optical depth spectrum to mark the Gaussians. Note the change to an optical depth spectrum with a changed color to indicate that a guess is needed.

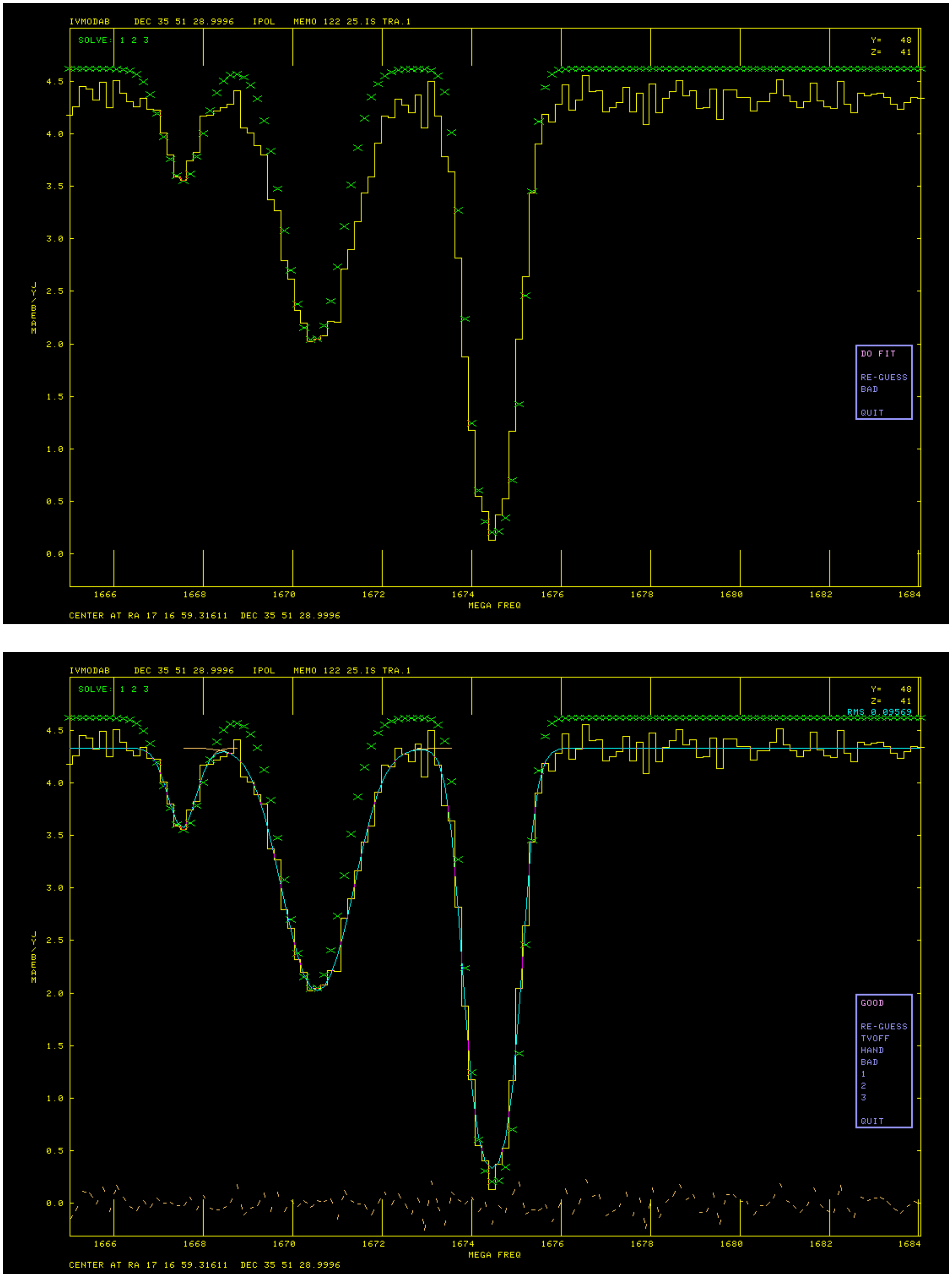


Figure 2: Top: First spectrum to fit: better initial "guess" has been entered so `DO FIT` should be selected. Bottom: First spectrum to fit: good fit obtained so `GOOD` or even `TVOFF` should be selected.

After the improved initial guess has been entered the plot is changed to show the new guess, as illustrated in Figure 2. This guess is good so the `DO FIT` option is selected. After TV "buttons" `A`, `B`, or `C` are pressed the task attempts the fit with the current initial guess. Then the plot is changed with the addition of the fit function in graphics plane 4 (usually cyan). This is illustrated in Figure 2. The individual Gaussians are also plotted in a graphics plane (7, usually a dirty yellow) which is only visible where no other graphics plane has been turned on.

A different menu appears at this point containing

`GOOD`	Mark solution good and go on
`DO FIT`	Attempt a new fit
`RE-GUESS`	Interactively enter a new initial guess
`TVOFF`	Mark solution good and turn off interactivity
`HAND`	Enter a solution from the terminal
`BAD`	Mark pixel bad and go on
`1`	Interactively enter a 1-Gaussian initial guess
`2`	Interactively enter a 2-Gaussian initial guess
`3`	Interactively enter a 3-Gaussian initial guess
`ENTER NG`	Interactively enter a number of Gaussians and an initial guess
`QUIT`	Exit `AGAUS` now

The option to `QUIT` causes the task to quit at this point. You may re-start later. The option `BAD` will mark this position as failed and go on to the next position. The option `RE-GUESS` will loop back to prompt you for a new guess. This takes you back through the steps shown in Figure 1 followed by that shown in Figure 2. Options `1`, `2`, `3`, `4`, `ENTER NG` will loop back to plot an initial guess with the selected number of Gaussians. The `ENTER NG` option appears only when the current `NGAUSS` is more than 4 and prompts you on the terminal to enter a number. (`NGAUSS=3` in the current example.) Option `HAND` will prompt you to enter using the terminal the Gaussian parameters for each component. Enter on one line for each component, the peak value of the Gaussian (in optical depth), the center (in pixels with respect to the reference pixel), and width (in pixels). Appropriate ranges of parameters in these units can be seen from the display of the current fit values. You may also enter flags to cause one or more parameter values to be fixed should you re-fit the current spectrum. The flags are entered after the 3 parameter values, flags ≤ 0 mean to hold the corresponding parameter fixed and omitted flags are taken as 1. `AGAUS` will then repeat the display in Figure 2 to see if you made a good guess. Immediately after a `HAND` operation only, the option `DO FIT` is offered to go back with the hand-entered values as the initial guess for a new fit.

Option `TVOFF` allows you to turn off interactivity, allowing the task to run using its own initial guesses until it finds a completely unreasonable solution or one with an rms greater than `RMSLIMIT`. When that happens, you are shown the offending fit parameters and the task resumes interactivity. Frequently this will drop you immediately into entering a new guess as in the bottom of Figure 1 where the color change should make this apparent. Sometimes it simply shows something like the top of Figure 1 and you must select `RE-GUESS` to try to fix things. Option `GOOD` tells the task that you are (reasonably) happy and that it should go on to the next position.

When fitting only a single Gaussian, `AGAUS` makes an initial guess based mostly on finding a real peak in the spectrum. This is quite reliable, so turning off the TV interaction may save a great deal of effort, although there will possibly be bad positions to be fixed up in the next stage of this task.

2.3 Editing and output

Eventually all positions selected by `FLUX` and `BLC` and `TRC` will have been fit. At this point, the task computes images of the fit parameters plus the "flux" (area under the Gaussian) and their uncertainties. An image of the rms over the first axis (after any fit components are subtracted) is also computed in the September `31DEC25` version (see Figure 3). This "edit" stage is omitted if the input image is one dimensional, but, in that

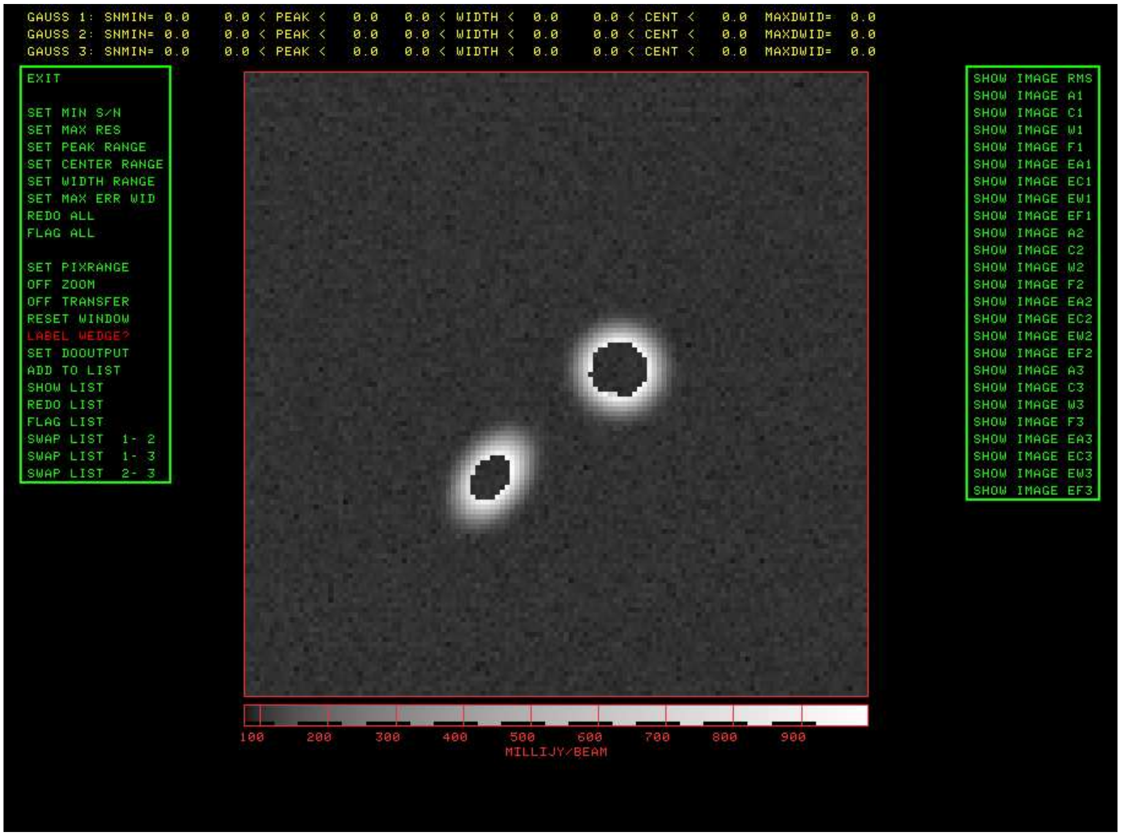


Figure 3: Model source residual rms image. The bright "rims" suggest that a lower FLUX cutoff would be useful.

case, channel flagging is available in the initial fitting of the spectrum. AGAUS then offers a lengthy menu of options which will allow you to view these images and revisit positions that seem to have produced incorrect fits. If NGAUSS > 1, options to swap portions of image n with corresponding portions of image m are also offered. This "edit" menu is illustrated in Figure 3. For small images, AGAUS replicates pixels in both directions to make the image large enough to see.

There are three kinds of editing implemented here. In the first, the user establishes the parameter extrema which should be viewed as acceptable. The extrema currently set are shown in the title lines. The SET options let you set the maximum residual and, for each Gaussian, the minimum signal-to-noise ratio, the minimum and maximum of the peak optical depth, the minimum and maximum of the center pixel, the minimum and maximum of the width, and the maximum error in the width. Then, AGAUS may be told to flag all solutions not meeting these criteria, or, more profitably perhaps, to revisit those positions to see if a better fit can be obtained. The other editing methods are similar, but act on a list of pixel positions. These may be entered by typing in values or by clicking on suspect pixel positions in the CURVALUE function described below or by selecting the BLOTCH LIST function also described below. The contents of the list may be viewed, the solutions at the positions may be flagged, or they may be revisited to attempt for a better solution, or the solutions at the listed positions may be swapped between components n and m. The menu will offer only appropriate swaps between components, thus 1, 2, and 3 in our NGAUSS=3 example. Swapping may be required if AGAUS gets confused as to which component you want to call number 1 and which number 2. After the flagging, revisiting, or swapping, the list is cleared. The first column of the menu shows the following:

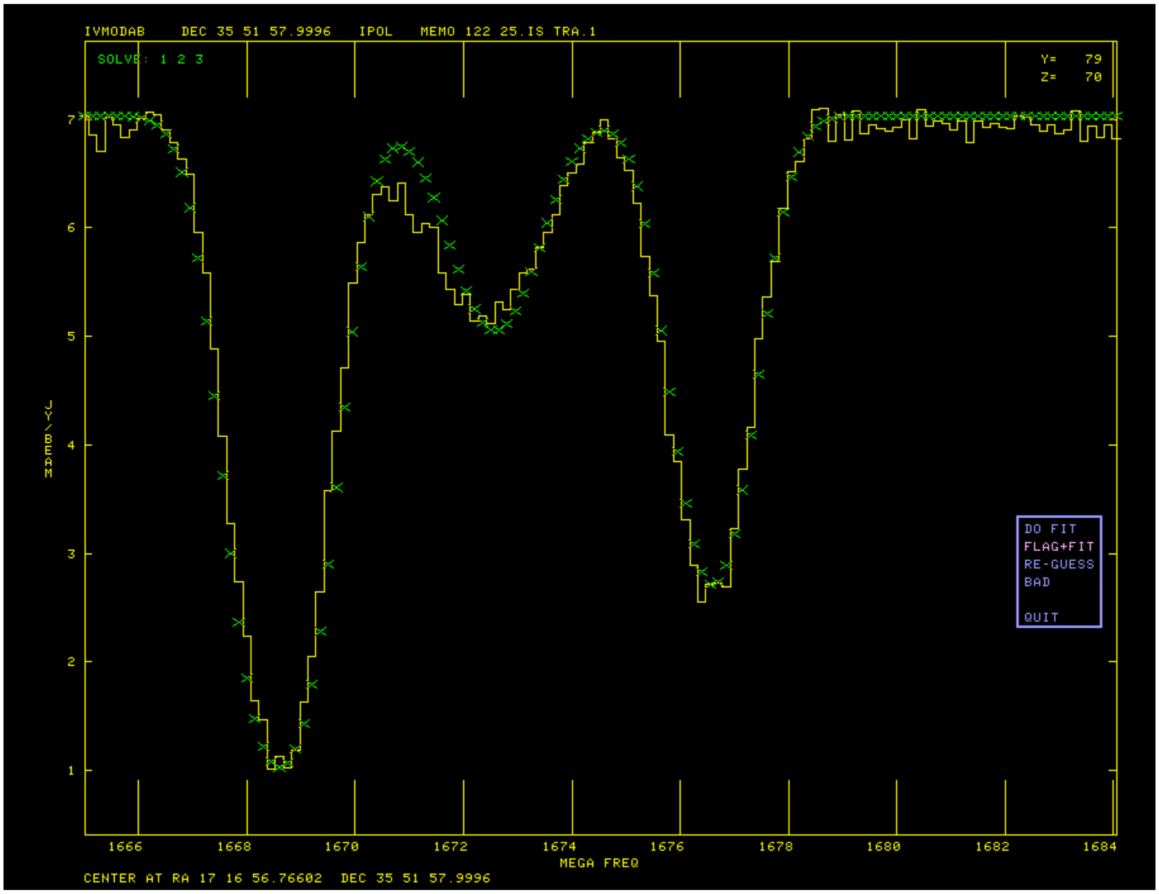


Figure 4: Spectrum to be re-done after doing a RE-GUESS. FLAG+FIT selected for illustrative purposes.

EXIT	Exit AGAUS, writing output images if DOOUTPUT is now > 0.
SET MIN S/N	Set minimum amplitude S/N(s) for okay solutions
SET MAX RES	Set maximum residual for okay solutions
SET PEAK RANGE	Set peak value range(s) for okay solutions
SET CENTER RANGE	Set X center range(s) for okay solutions
SET WIDTH RANGE	Set width range(s) for okay solutions
SET MAX ERR WID	Set maximum error(s) in width for okay solutions
REDO ALL	Re-do all solutions which are not okay following the above criteria
FLAG ALL	Mark bad all solutions which are not okay
OFF ZOOM	Turn of TV zoom
OFF TRANSFER	Turn off black & white and color TV enhancements
RESET WINDOW	Display full view of current image
LABEL WEDGE?	Turn on/off labeling of step wedge
SET DOOUTPUT	Increment DOOUTPUT in loop 0-3 — with 1 and 3 causing residual images and 2 and 3 causing parameter images to be written on EXIT
ADD TO LIST	Type in output pixel coordinates to add to list
SHOW LIST	Display coordinates in list
REDO LIST	Re-do solutions for all pixels in list
FLAG LIST	Flag solutions for all pixels in list
SWAP LIST 1-2	Swap solutions for components 1 and 2 for all pixels in list
SWAP LIST 1-3	Swap solutions for components 1 and 3 for all pixels in list
SWAP LIST 2-3	Swap solutions for components 2 and 3 for all pixels in list

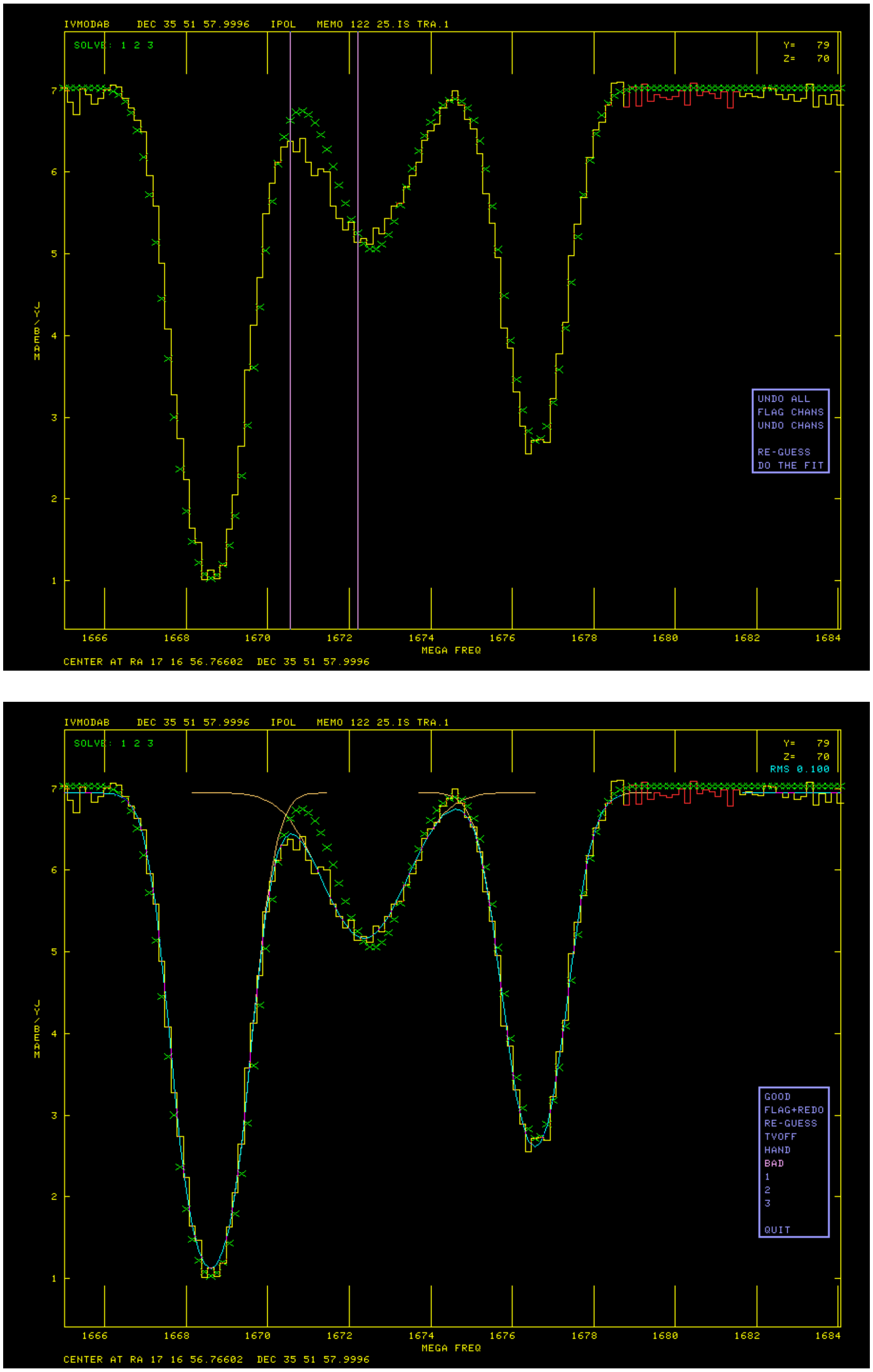


Figure 5: Top:Spectrum to be re-done in flag menu. Some channels on the right hand side are flagged and the pink lines show another range of channels which could be flagged. Bottom: Acceptance menu after `FLAG+FIT`.

In 31DEC25, AGAUS offers an additional capability in the edit stage.[3] When doing REDO ALL and REDO LIST, the menu shown in Figure 1 and Figure 2 has an additional option FLAG+FIT as shown in Figure 4. When this option is selected a new menu is shown as in Figure 5:

UNDO ALL	Undo all channel flagging
FLAG CHANS	Interactively flag a range of channels
UNDO CHANS	Interactively undo flags in a range of channels
RE-GUESS	Interactively enter a new guess for the Gaussians
DO THE FIT	Leave the flag routine and do the fit

Flagged channels are plotted in a different color (usually reddish) than unflagged channels (usually yellow). UNDO ALL simply removes all flags in the current spectrum. FLAG CHANS and UNDO CHANS bring up two vertical lines (usually pink in color). The cursor position controls the position of the lines, button A switches between the two, button B sets a channel range and continues, button C sets a range and exits, and button D exits without setting a new range. RE-GUESS takes you to the screen to set a new guess (Figure 1) and then returns to this flag menu. Finally DO THE FIT attempts the Gaussian fit and returns to the menu to accept the result. That screen is illustrated in the bottom of Figure 5. Note the FLAG+REDO option added to this menu in the edit phase. That option takes you back through the sequence just illustrated.

When the XG table is created with fewer than six Gaussians, the second column of the main edit menu contains the rms image plus NGAUSS sets of functions

SHOW IMAGE RMS	Enter image interaction with the residual rms image
SHOW IMAGE A1	Enter image interaction with peak value of component 1
SHOW IMAGE C1	Enter image interaction with center pixel of component 1
SHOW IMAGE W1	Enter image interaction with width of component 1
SHOW IMAGE F1	Enter image interaction with "flux" of component 1
SHOW IMAGE EA1	Enter image interaction with uncertainty in peak value of component 1
SHOW IMAGE EC1	Enter image interaction with uncertainty in center pixel of component 1
SHOW IMAGE EW1	Enter image interaction with uncertainty in width of component 1
SHOW IMAGE EF1	Enter image interaction with uncertainty in "flux" of component 1

When the XG table is created with more than 5 components, these menus change as illustrated in Figure 6. The SWAP LIST option lists all possible swaps for the first 5 Gaussians. The added option SWAP LIST M - N prompts the user on the terminal for the two components to be switched using the pixel list. The right-hand column changes significantly. Only one component can be viewed at any one time, with the CHOOSE COMP nn option to select the desired component. For this figure, the user has looked at the rms image and then selected Gaussian component 4 to be examined. The right-hand menu becomes

SHOW IMAGE RMS	Enter image interaction with the residual rms image
SHOW IMAGE A4	Enter image interaction with peak value of component 4
SHOW IMAGE C4	Enter image interaction with center pixel of component 4
SHOW IMAGE W4	Enter image interaction with width of component 4
SHOW IMAGE F4	Enter image interaction with "flux" of component 4
SHOW IMAGE EA4	Enter image interaction with uncertainty in peak value of component 4
SHOW IMAGE EC4	Enter image interaction with uncertainty in center pixel of component 4
SHOW IMAGE EW4	Enter image interaction with uncertainty in width of component 4
SHOW IMAGE EF4	Enter image interaction with uncertainty in "flux" of component 4
CHOOSE COMP 1	Change attention to Gaussian component 1.
CHOOSE COMP 2	Change attention to Gaussian component 2.
CHOOSE COMP 3	Change attention to Gaussian component 3.
CHOOSE COMP 4	Change attention to Gaussian component 4.
CHOOSE COMP 5	Change attention to Gaussian component 5.
CHOOSE COMP 6	Change attention to Gaussian component 6.

[3] This option is available at the beginning when the input image is one-dimensional.

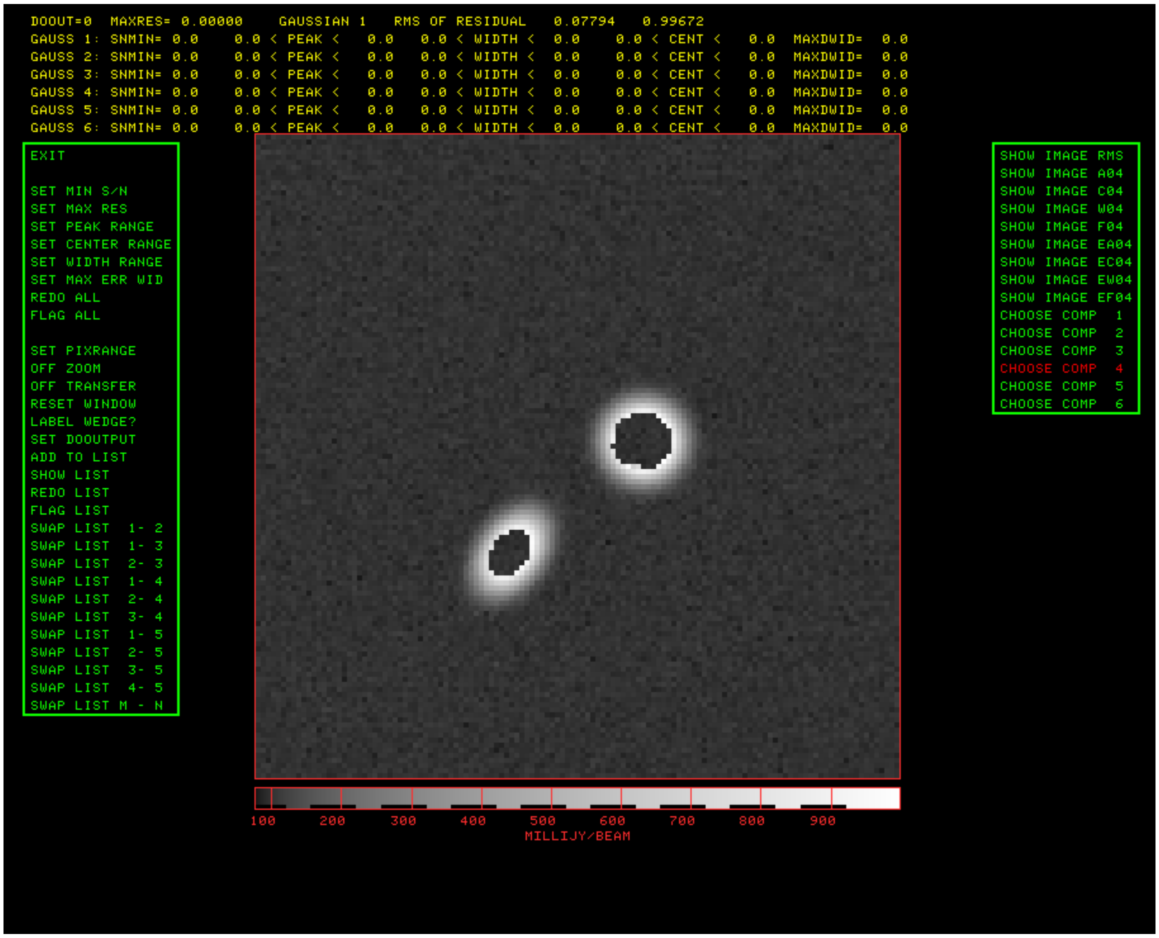


Figure 6: Edit menu when more than 5 components. The rms image is shown with component 4 selected.

When you select one of the `SHOW IMAGE` functions most of the following operations will appear in yet another menu. This image interaction menu is illustrated in Figure 7. Only one of the `LOAD AS` options will appear, with the next one in the sequence offered when the current one has been invoked. The `SET WINDOW` option allows you to select a sub-image to view in greater detail, while `RESET WINDOW` returns to viewing the full image. The `BLOTCH LIST` option allows you to set an area of the image to be added to the list of pixels to be addressed later with `REDO LIST`, `FLAG LIST`, or SWAP LIST.. The `SWAP` n–m options will appear as needed when `NGAUSS` > 1 and are used to reverse your selection of component number n and component m in case `AGAUS` got confused. The `NEXT WINDOW` option appears when needed to display an image too large to fit on the display screen. These options mostly invoke familiar functions from *AIPS* to control the `FUNCTYPE` used in loading the image to the display, to enhance the image intensities, to color the enhanced image intensities, and to zoom the display.

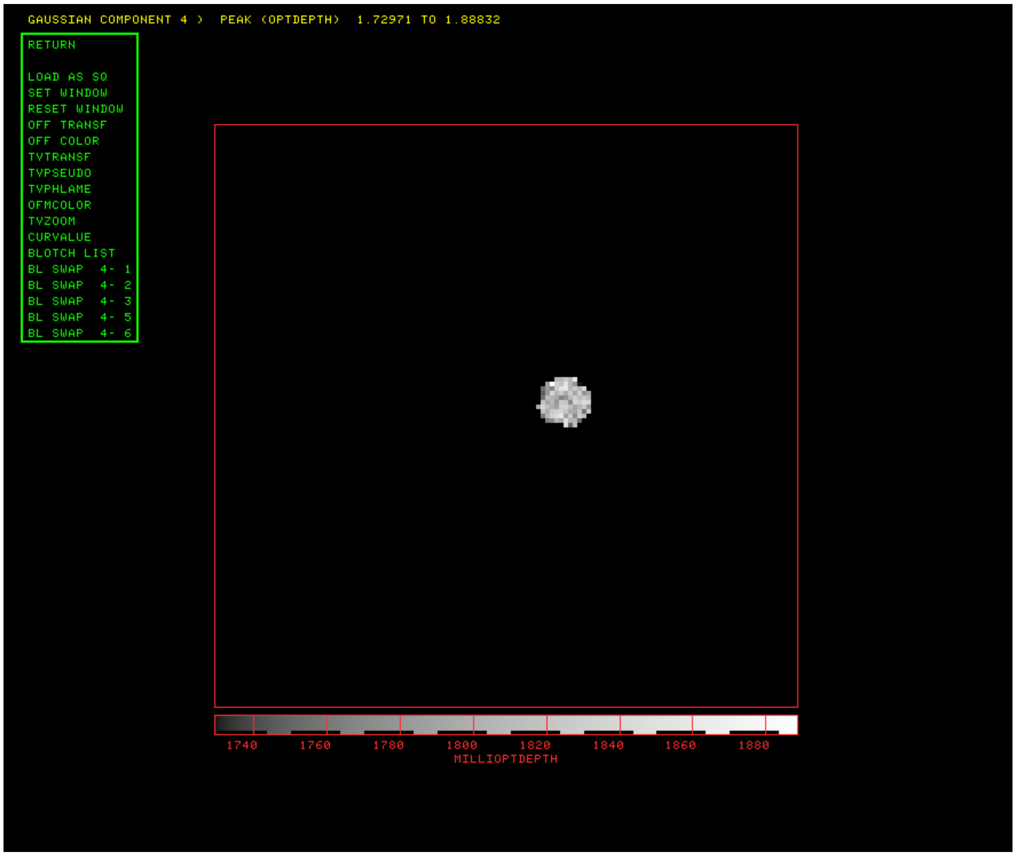


Figure 7: Image of model Gaussian component 4 when `NGAUSS` = 6.

`RETURN`	Return to the above menus, image stays displayed
`LOAD AS SQ`	Re-load image with square root transfer function
`LOAD AS LG`	Re-load image with log transfer function
`LOAD AS L2`	Re-load image with extreme log transfer function
`LOAD AS LN`	Re-load image with linear transfer function
`SET WINDOW`	Set a sub-image to view
`RESET WINDOW`	Return too viewing the full image
`OFF TRANSF`	Turn off enhancement done with `TVTRANSF`
`OFF COLOR`	Turn off color enhancements
`TVTRANSF`	Black & white image enhancement
`TVPSEUDO`	Color enhancement of numerous sorts
`TVPHLAME`	Color enhancement of flame type, multiple colors
`OFMCOLOR`	Many standard color tables (OFMs) available
`TVZOOM`	Interactive zooming and centering of image
`CURVALUE`	Display value under cursor, mark pixels for list
`BLOTCH LIST`	Interactively add a blotch area to the pixel list
`SWAP 1-2`	Swap solutions for components 1 and 2 interactively
`SWAP 1-3`	Swap solutions for components 1 and 3 interactively
`NEXT WINDOW`	Move to next window in large images

Two operations in this menu are different from the usual. `CURVALUE` provides the capability of selecting positions for the edit "list." During the `CURVALUE` operation position the cursor over the desired pixel and press buttons `A`, `B`, or `C` to add that pixel to the list. The `BLOTCH LIST` and `SWAP` n-m operations use a TV blotch operation like that in the *AIPS* verb TVSTAT and task BLSUM. You are to mark with a "blotch" region those pixels in the present image which are to be added to the list or are to have their solutions swapped with those of the selected component. Instructions will appear in the message window as you proceed. Begin by positioning the cursor at a pixel to be the first vertex of a connected sequence of vertices and press TV button A. Move to the next vertex and press button A again and repeat until you have marked all vertices for this region. Then press button D if you are done with this region or button C if you need to re-position one of the vertices. In this case, move the cursor to the vertex to be re-positioned, press button A and drag the vertex to the corrected position. Press button A or B to fix that vertex and go on to reset another vertex or D to end this region. You may do this as many times as needed.

When you have finished getting the images just the way you want them, you may write them out as *AIPS* image files. Select the SET DOOUTPUT option until its value, shown at the top of the screen, is what you want. In AGAUS, values 1, 3, 5, and 7 cause a residual image cube to be written, values 2, 3, 6 and 7 cause images of the parameter values and their uncertainties to be written, and values 4, 5, 6, and 7 cause the image of the residual rms to be written. The baseline and slope images and their uncertainties are given class CONST, SLOPE, DCONST, and DSLOPE, while the parameter images and their uncertainties get class AMPLnn, CENTnn, WIDTnn, DAMPnn, DCENnn, and DWIDnn, and the flux and its uncertainty get class FLUXnn, and DFLXnn, where nn is the component number. The residual rms image gets class RESRMS.

3 Zeeman splitting: ZAMAN

When an intrinsically unpolarized spectral line is emitted in the presence of a magnetic field, the right and left circular polarizations have their frequencies shifted in opposite directions by an amount proportional to the magnetic field (at least for modest magnetic fields). The traditional analysis of data to measure this splitting works only for those cases in which the separation of polarizations is a small fraction of the line width. In that case, the function that has been traditional is

$$V(x,y,z) = A(y,z)I(x,y,z) + 0.5B(y,z)\,\frac{dI(x,y,z)}{dx} \tag{3}$$

where $V(x,y,z)$ is the V Stokes polarization component. $I(x,y,z)$ is the unpolarized I Stokes component, x is the spectral axis value, (y,z) is the celestial coordinate value, and $A(y,z)$ and $B(y,z)$ are the parameters to be found by a linear least-squares method. [4] The *AIPS* task ZAMAN offers this model, with two choices for the method by which the derivative of I with x is computed.

If the total intensity spectrum has had AGAUS applied, another function may be used instead. I present below a derivation of the formula for absorption, neglecting the leakage term until the end. Let us assume that the optical depth is due to a sum of Gaussians each having its own depth, center channel, and width. Thus

$$\begin{aligned} I(x,y,z) &= C(y,z)exp(-\sum_{1}^{\mathtt{NGAUSS}} \tau_i(x,y,z)) && (4)\\ &= C(y,z)\prod_{1}^{\mathtt{NGAUSS}} exp(-\tau_i(x,y,z)) && (5)\end{aligned}$$

When shifted by magnetic fields, assumed different for each Gaussian, the formula for the V Stokes becomes

$$\begin{aligned} V(x,y,z) &\equiv C(y,z)\left(\prod_{1}^{\mathtt{NGAUSS}} exp(-\tau_{iR}(x,y,z)) - \prod_{1}^{\mathtt{NGAUSS}} exp(-\tau_{iL}(x,y,z))\right) && (6)\\ &= C(y,z)\prod_{1}^{\mathtt{NGAUSS}} exp(-\tau_{iA})exp(-(\tau_{iR}-\tau_{iA})) - \prod_{1}^{\mathtt{NGAUSS}} exp(-\tau_{iA})exp(-(\tau_{iL}-\tau_{iA})) && (7)\end{aligned}$$

[4] Sault, R.J., Killeen, N. E. B.., Zmuidzinas, J., Loushin, R. 1990, *Ap. J.*, **74**, 437-461.

where we assume that the difference between τ_{iL} and τ_{iA} is small. Then

$$\begin{aligned}
exp(-(\tau_{iL}-\tau_{iA})) &\approx 1+\tau_{iA}-\tau_{iL} && (8)\\
&= 1+\tau_{i0}\left(\exp(-\alpha(x-x_i)^2/\sigma_i^2)-\exp(-\alpha(x-(x_i+\delta_i/2))^2/\sigma_i^2)\right) && (9)\\
&\approx 1+\tau_{i0}\exp(-\alpha(x-x_i)^2/\sigma_i^2)\left[1-exp(\alpha(x-x_i)\delta_i/\sigma_i^2-\alpha\delta_i^2/(4\sigma_i^2))\right] && (10)\\
&\approx 1+\tau_{i0}\exp(-\alpha(x-x_i)^2/\sigma_i^2)\left[-\alpha(x-x_i)\delta_i/\sigma_i^2\right] && (11)\\
exp(-(\tau_{iR}-\tau_{iA})) &\approx 1+\tau_{i0}\exp(-\alpha(x-x_i)^2/\sigma_i^2)\left[\alpha(x-x_i)\delta_i/\sigma_i^2\right] && (12)
\end{aligned}$$

Thus each of the terms in Equation 7 is of the form

$$\begin{aligned}
&= C(y,z)\prod_{1}^{\texttt{NGAUSS}} exp(-\tau_{iA})(1\pm a_i) && (13)\\
&= C(y,z)exp(-\sum\tau_i)\prod_{1}^{\texttt{NGAUSS}}(1\pm a_i) && (14)\\
&\approx C(y,z)exp(-\tau)\left(1\pm\sum a_i\right) && (15)
\end{aligned}$$

We note that the product of the $\exp(-\tau_{iA})$ is just the total absorption factor $\exp(-\tau)$ and the product of a number of $(1\pm a_i)$, where a_i is small, is approximately $1\pm\sum a_i$. We end up with

$$\begin{aligned}
\tau(x,y,z) &= \sum_{1}^{\texttt{NGAUSS}}\tau_{i0}\exp(-\alpha(x-x_i)^2/\sigma_1^2) && (16)\\
V(x,y,z) &\approx C(y,z)exp(-\tau(x,y,z))\sum_{1}^{\texttt{NGAUSS}}(2\tau_i\alpha(x-x_i)\delta_i/\sigma_i^2)exp(-\tau_i(x,y,z)) && (17)\\
V(x,y,z) &= A(y,z)I(x,y,z)+0.5\sum_{1}^{\texttt{NGAUSS}}B_i(y,z)\,\frac{-d\tau_i(x,y,z)}{dx} && (18)
\end{aligned}$$

where $\tau_i(x,y,z)$ is the spectrum of the i'th Gaussian component in the absorption and one solves for the I polarization leakage $A(y,z)$ and `NGAUSS` component frequency separations $B_i(y,z)$. Any association of $B_i(y,z)$ with magnetic field values is left to the user.

3.1 Inputs

The inputs for `ZAMAN` are very similar to those for `AGAUS`. `INNAME` *et al.* specify the V polarization cube which must be in transposed form with frequency/velocity as the first axis. `IN2NAME` *et al.* specify the corresponding I polarization cube, similarly transposed. The axes of the two images must match. `BLC` and `TRC` define the spectral and celestial coordinate regions of interest for the fit and `FLUX` gives the lower limit in the I image for the average of three consecutive channels for the spectra to be fit. `INVERS` specifies the input version of the `ZE` table in which the results are stored. Zero means to make a new one, other values mean to re-visit an existing solution. Adverb `OPTYPE` specifies which of the above formulæ are solved. `OPTYPE = 'GAUS'` says to use the `XG` file `IN2VERS` attached to the input I polarization cube to solve using Equation 17. `OPTYPE = '2SID'` says to solve Equation 3, evaluating the derivative by

$$\frac{dI(x,y,z)}{dx}=0.5\,[\,I(x+1,y,z)-I(x-1,y,z)\,]$$

while any other `OPTYPE` value says to evaluate the derivative with

$$\frac{dI(x,y,z)}{dx}=I(x,y,z)-I(x-1,y,z)$$

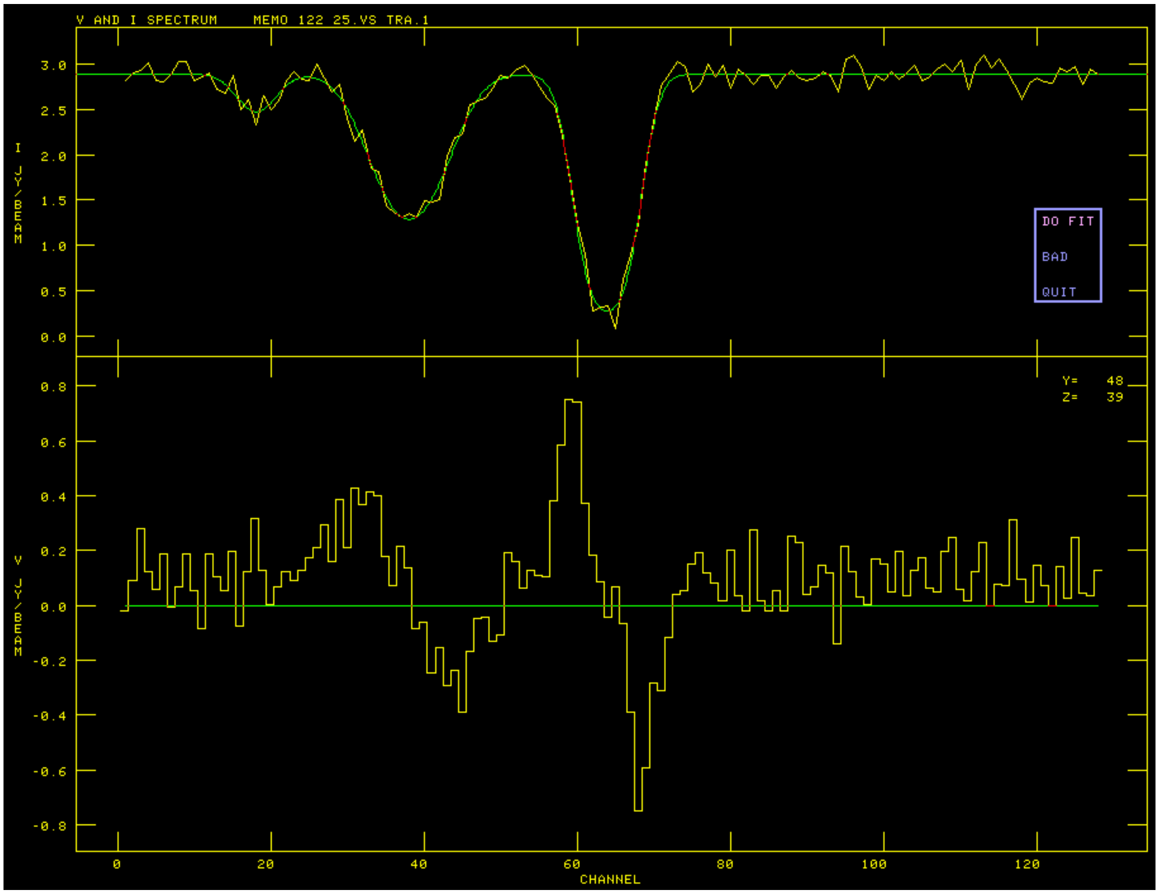


Figure 8: First model spectrum to fit, initial "guess" has Gaussians but the two $B_i(y,z)$ are zero.

while solving Equation 3. DOOUTPUT controls what files are written — this may be changed interactively (except for one-dimensional images) so leave it zero at this point. Set DOTV = 1 to use TV menus to prompt you. Even though the fitting operation is linear and so needs little or no guidance, it is best to watch what is happening so you should never set this adverb false. If all seems well, you can turn off the TV after watching a few of the solutions. $A(y,z)$ is always fit, but should be vanishingly small if ZAMAN is applied to a cube written by ZAMAN with the leakage removed. RMSLIMIT is an upper limit for the rms of a fit before the fit is viewed as "failed" which causes the TV and interaction to be turned back on after you have turned it off. You should get a good idea of an appropriate value from your initial uses of ZAMAN or from your knowledge of the noise in your data cube.

ZAMAN has one additional adverb, DOCONT, for which the special value -2 causes changed behavior in the fitting. (A special value was chosen to avoid accidental use of this option since DOCONT is used by a number of other tasks.) If the Vpol image cube was produced from continuum-subtracted visibilities (*i.e.*, the output of UVLSF or UVLIN), then the actual Ipol leakage will be proportional to the continuum subtracted Ipol, $I(x,y,z) - C(y,z)$ rather than $I(x,y,z)$. ZAMAN does the fitting differently when this option is selected and records the fact in the ZE table by setting the keyword ABSORPTN to 2, rather than the usual 1 for normal absorption modeling. The post-fitting plot task XG2PL uses this keyword value to honor the changed meaning of the leakage gain.

3.2 Fitting

The fitting process starts with a plot of the I polarization spectrum across the top and the V polarization across the bottom. The data and axis labels are plotted in graphics channel one (usually yellow) and the initial guess as lines in graphics channel two (usually green). The Gaussians, when `OPTYPE='GAUS'`, are shown as a smooth line on the I polarization plot, while the initial guess, plotted in the V polarization, is plotted at the locations of the data samples. Then you are offered a menu of options, as shown in the present figures, on the TV (`DOTV`> 0). The first spectrum to be fit (from the model of the absorption shown for `AGAUS`) is illustrated in Figure 8 showing that the first guess has zero for the three $B_i(y, z)$ and also, in this case, $A(y, z)$. With a good guess for the gain, the green line can mimic the shape of the I spectrum, not the V spectrum. The menu that appears at this point is

`DO FIT`	To do the fitting
`BAD`	To mark this voxel bad and go on to the next
`QUIT`	To exit `ZAMAN` now

Note that there is no `RE-GUESS` since there is no need to enter them with linear least squares. You select menu options in the same way as `AGAUS`, with "button" `D` giving appropriate real-time help information and buttons `A`, `B`, and `C` selecting the highlighted option. The option to `QUIT` causes the task to quit at this point. You may re-start later. The option `BAD` will mark this position as failed and go on to the next position. The option `DO FIT`, currently highlighted, will cause the task to attempt the linear fit with the current initial guess. The result of that fit is shown in the top Figure 9. The menu at this point contains

`GOOD`	To accept the solution and go on
`TVOFF`	To accept the solution and go on with no interaction
`HAND`	To enter a new solution from the terminal
`BAD`	To mark this solution as bad and go on
`QUIT`	To exit `ZAMAN` now

What most of these do is obvious. `HAND` prompts you to enter the Gain value and then either *Enter avg Field in channels* or *Enter Gaussian xx Field in channels* for $xx = 1$ to `NGAUSS` depending on the `OPTYPE` selected. When you have entered all values, `ZAMAN` repeats the plot at the top of Figure 9 with the new values. When the task goes on to the next voxel it starts with the $B_i(y, z)$ of the last fit (bottom of Figure 9) and so has a much better initial guess.

3.3 Editing and output

Eventually all positions selected by `FLUX` and `BLC` and `TRC` will have been fit. At this point, the task computes images of residual spectral rms and the fit parameters $A(y, z)$ and $B_i(y, z)$ and their uncertainties. This "edit" stage is omitted if the input image is one dimensional, but, in that case, channel flagging is available in the initial fitting of the spectrum. `ZAMAN` then offers a menu of options which will allow you to view these images and revisit positions that seem to have produced incorrect fits. This "edit" menu is illustrated in Figure 10. If `SET WINDOW` is used to select a small sub-image, then `ZAMAN` will blow it up by pixel replication to a reasonable size as in the figure. The first column of the menu includes:

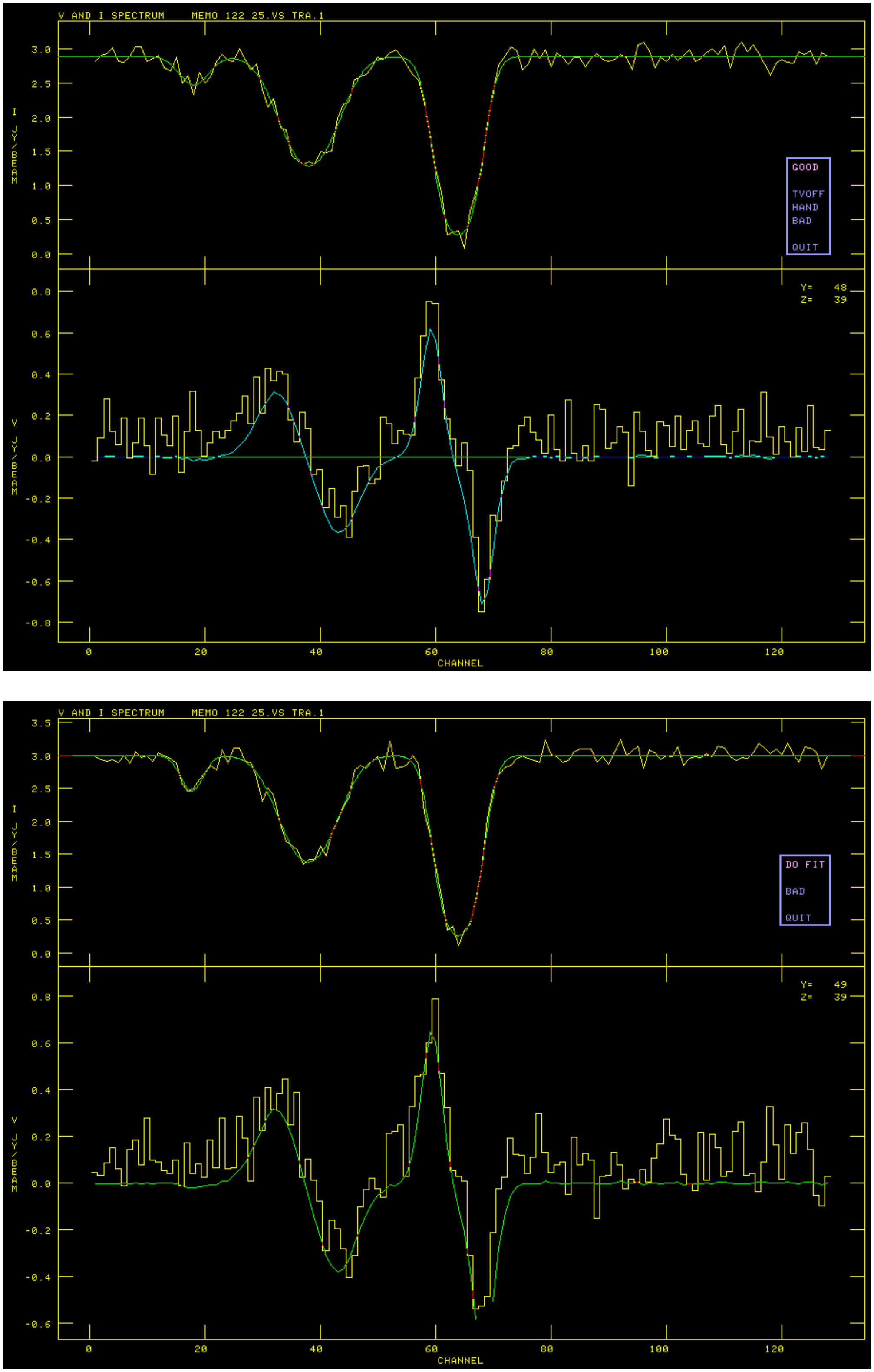


Figure 9: Top: First model spectrum to fit plotted after fitting. Bottom: Next model spectrum, initial "guess" in green has Gaussians and values for $B_i(y,z)$, fit is in cyan.

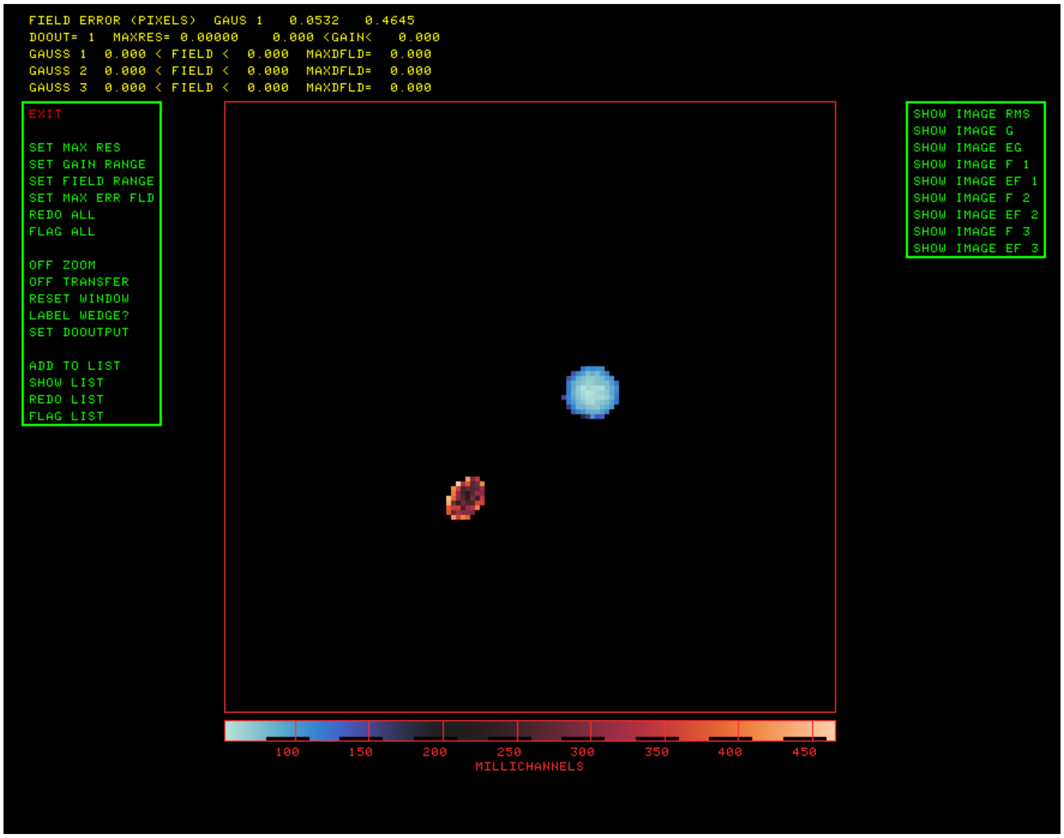


Figure 10: `ZAMAN` fit and image editing screen; image shown is the uncertainty in the field of component 1. Coloring from `OFMCOLOR` of button C.

+

`EXIT`	Exit `AGAUS`, writing output images if `DOOUTPUT` is now > 0.
`SET MAX RES`	Set maximum residual for okay solutions
`SET GAIN RANGE`	Set gain value range(s) for okay solutions
`SET FIELD RANGE`	Set field range(s) for okay solutions
`SET MAX ERR FLD`	Set maximum error(s) in field for okay solutions
`REDO ALL`	Re-do all solutions which are not okay
`FLAG ALL`	Mark bad all solutions which are not okay
`OFF ZOOM`	Turn of TV zoom
`OFF TRANSFER`	Turn off black & white and color TV enhancements
`RESET WINDOW`	Display full view of current image
`LABEL WEDGE?`	Turn on/off labeling of step wedge
`SET DOOUTPUT`	Increment `DOOUTPUT` in loop 0-3 — with 1 and 3 causing residual images and 2 and 3 causing parameter images to be written on `EXIT`
`ADD TO LIST`	Type in output pixel coordinates to add to list
`SHOW LIST`	Display coordinates in list
`REDO LIST`	Re-do solutions for all pixels in list
`FLAG LIST`	Flag solutions for all pixels in list

There are two kinds of editing implemented here. In the first, the user establishes the parameter extrema which should be viewed as acceptable. The extrema currently set are shown in the title lines and include the maximum rms, the range of allowed values for "gain" ($A(y, z)$), the range of allowed values for "field" ($B(y, z)$ in pixels), and the maximum uncertainty in the field. Then, `ZAMAN` may be told to flag all solutions

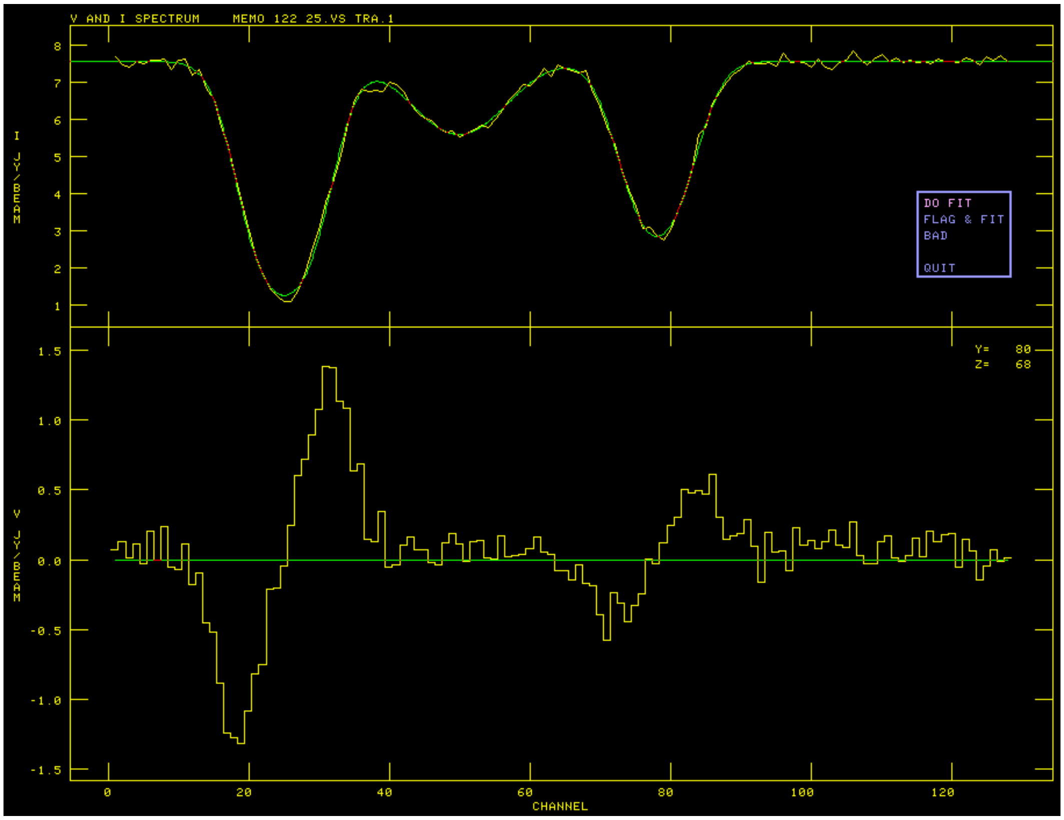


Figure 11: `ZAMAN` initial display to re-do a fit.

not meeting these criteria, or, more profitably perhaps, to revisit those positions to see why a poor fit was obtained. Note that, unlike `AGAUS`, the linear nature of the fit in `ZAMAN` means that only one solution is possible at each celestial coordinate without a change to the Gaussian model or to the data of the spectrum. The other editing method is similar, but acts on a list of pixel positions. These may be entered by typing in values, by clicking on suspect pixel positions in the `CURVALUE` function described below, or by the `BLOTCH LIST` function also described below. The contents of the list may be viewed, the solutions at the positions may be flagged, and they may be revisited to see why they are suspect. Note that there are no swapping of solutions in this task; `AGAUS` establishes which component is which. After the flagging or revisiting, the list is cleared.

In `31DEC25` the operations of `REDO ALL` and `REDO LIST` offer an additional menu option.[5] When a spectrum to be re-done is first displayed it is called `FLAG & FIT` as illustrated in Figure 11. If this option is selected the screen shown in the upper portion of Figure 12 appears. The menu contains

`UNDO ALL`	Undo all channel flagging
`FLAG CHANS`	Interactively flag a range of channels
`UNDO CHANS`	Interactively undo flags in a range of channels
`DO THE FIT`	Leave the flag routine and do the fit

Flagged channels are plotted in a different color than unflagged channels (usually reddish and yellow, respectively). `UNDO ALL` simply removes all flags on the current spectrum. `FLAG CHANS` and `UNDO CHANS` bring up two vertical lines (usually pink in color). The cursor position controls the position of the lines,

[5] This option is available at the beginning when the input image is one-dimensional.

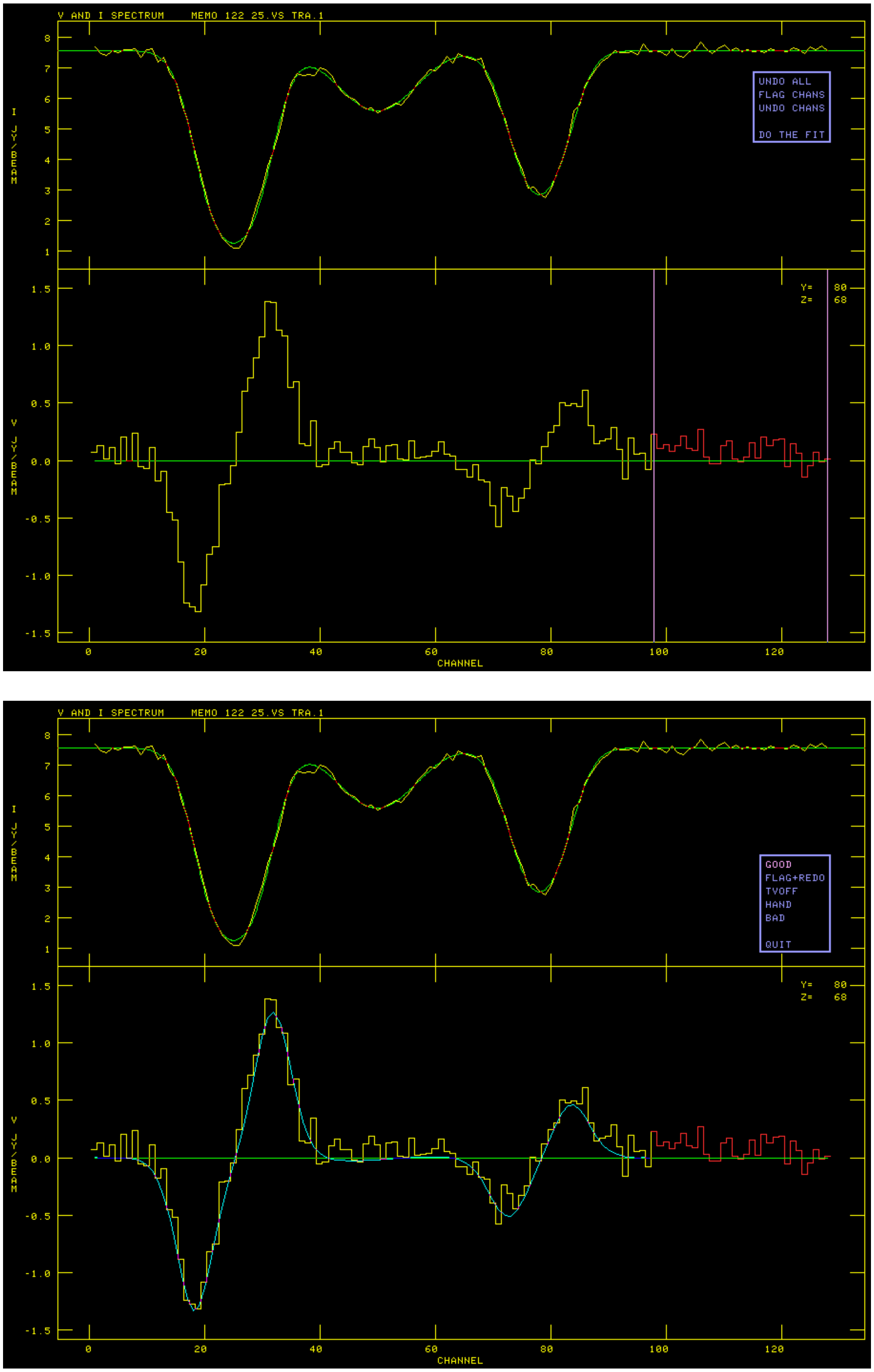


Figure 12: Top: Spectrum to be re-done in flag menu. Some channels on the right hand side are flagged and the pink lines show another range of channels which could be flagged. Bottom: Acceptance menu after `FLAG & FIT`.

button A switches between the two, button B sets a channel range and continues, button C sets a range and exits, and button D exits without setting a new range. Finally `DO THE FIT` attempts the Zeeman fit and returns to the menu to accept the result. That screen is illustrated in the bottom of Figure 12. Note the `FLAG+REDO` option added to this menu in the edit phase. That option takes you back through the sequence just illustrated.

The second column of the main edit menu contains

`SHOW IMAGE G`	Enter image interaction with gain
`SHOW IMAGE EG`	Enter image interaction with uncertainty in the gain
`SHOW IMAGE F1`	Enter image interaction with field of component 1
`SHOW IMAGE EF1`	Enter image interaction with uncertainty in field of component 1
`SHOW IMAGE F2`	Enter image interaction with field of component 2
`SHOW IMAGE EF2`	Enter image interaction with uncertainty in field of component 2
`SHOW IMAGE F3`	Enter image interaction with field of component 3
`SHOW IMAGE EF3`	Enter image interaction with uncertainty in field of component 3

There is 1 or, if `OPTYPE='GAUS'` as in the illustrated cases, the maximum of `NGAUSS` of the Fn and EFn options. When you select one of the `SHOW` options, the options in yet another menu appear along with a display of the selected image. As illustrated in Figure 13, these options are

`RETURN`	Return to the above menus, image stays displayed
`LOAD AS SQ`	Re-load image with square root transfer function
`LOAD AS LG`	Re-load image with log transfer function
`LOAD AS L2`	Re-load image with extreme log transfer function
`LOAD AS LN`	Re-load image with linear transfer function
`SET WINDOW`	Set a sub-image to view
`RESET WINDOW`	Return too viewing the full image
`OFF TRANSF`	Turn off enhancement done with `TVTRANSF`
`OFF COLOR`	Turn off color enhancements
`TVTRANSF`	Black & white image enhancement
`TVPSEUDO`	Color enhancement of numerous sorts
`TVPHLAME`	Color enhancement of flame type, multiple colors
`OFMCOLOR`	Many standard color tables (OFMs) available
`TVZOOM`	Interactive zooming and centering of image
`CURVALUE`	Display value under cursor, mark pixels for list
`BLOTCH LIST`	Interactively add a blotch area to the pixel list
`NEXT WINDOW`	Move to next window into large images

Only one of the `LOAD AS` options will appear, namely the next after the current transfer function from the list of linear, square root, log, and more extreme log transfer functions. Most of the options are familiar as `AIPS` verbs. Instructions for interaction will appear on the terminal and button `D` in the menu may be used to obtain a helpful display on the terminal.

`CURVALUE` provides the capability of selecting positions for the edit "list." During the `CURVALUE` operation position the cursor over the desired pixel and press buttons `A`, `B`, or `C` to add that pixel to the list. The `BLOTCH LIST` operation uses a TV blotch operation like that in the *AIPS* verb `TVSTAT` and task `BLSUM`. You are to mark with a "blotch" region those pixels in the present image which are to be added to the list. Instructions will appear in the message window as you proceed. Begin by positioning the cursor at a pixel to be the first vertex of a connected sequence of vertices and press TV button `A`. Move to the next vertex and press button `A` again and repeat until you have marked all vertices for this region. Then press button `D` if you are done with this region or button `C` if you need to re-position one of the vertices. In this case, move the cursor to the vertex to be re-positioned, press button `A` and drag the vertex to the corrected position. Press button `A` or `B` to fix that vertex and go on to reset another vertex or `D` to end this region. You may do this as many times as needed.

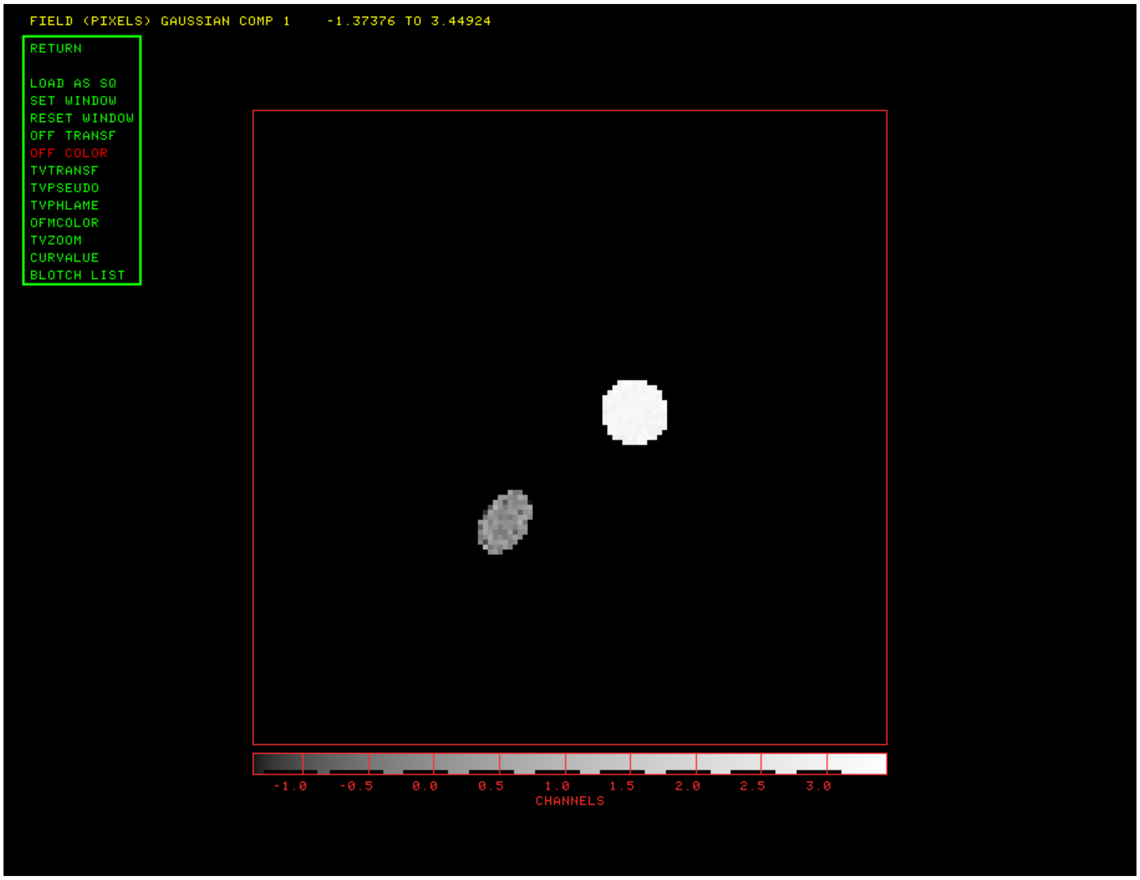


Figure 13: ZAMAN image of field of component 1 ($B_1(y, z)$) with display options.

When you have finished getting the images just the way you want them, you may write them out as AIPS image files. Select the SET DOOUTPUT option until its value, shown at the top of the screen, is what you want. In ZEMAN, values 4, 5, 6, 7, 12, 13, 14, and 15 cause a residual (data-model) image cube to be written, values 2, 3, 6, 7, 10, 11, 14, and 15 cause images of the gain and field values and their uncertainties to be written, values 1, 3, 5, 7, 9, 11, 13 and 15 cause an image of the V cube to be written with the gain times the I-polarization cube subtracted, and finally values greater than 7 cause an image of the residual rms to be written. The corrected V-polarization image gets the class specified by OUTCLASS, the residual image gets class VRESID, the gain and its uncertainty get classes GAIN and DGAIN, and the field(s) and their uncertainties get classes FELDnBn and DFELnn, with $nn = 1$ to the maximum NGAUSS. The residual rms image gets class RESRMS.

4 Post-fit plotting

The images produced by XGAUS, AGAUS, ZEMAN and ZAMAN may be displayed using all the usual tools such as KNTR, PROFL, and numerous other tasks. However, the display of the spectral data and the various fits to them required a new task.

XG2PL plots a spectrum for a single pixel or for a rectangular or circular region about a single pixel. For each pixel included in the average, the task reads the I polarization image to obtain the data and the appropriate row in the XG table to obtain the AGAUS solution for that pixel. It then computes the spectrum of each component in the model, plus the sum of the components, and the residual (data-model). Each of these then

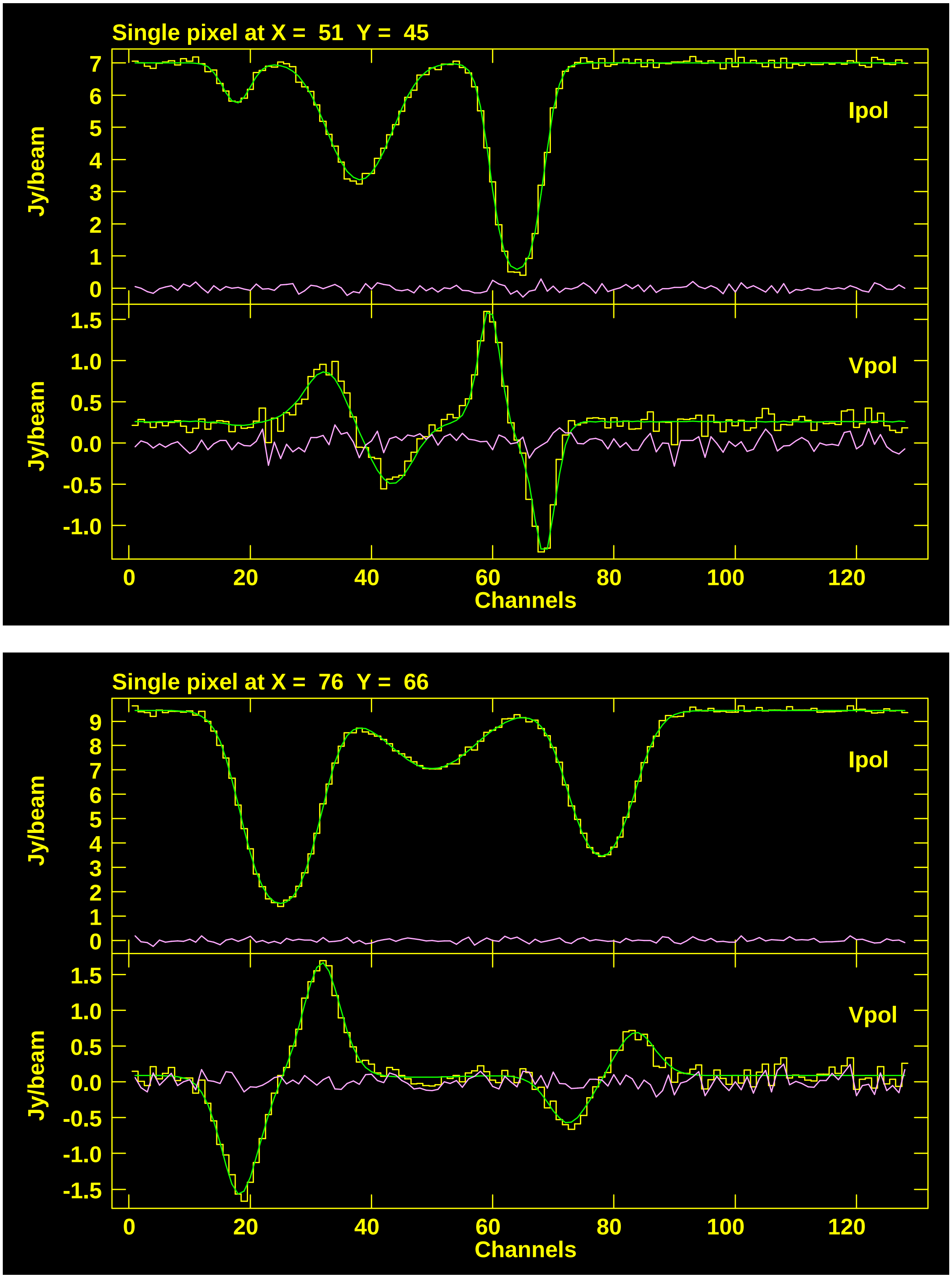


Figure 14: Example spectra from two single pixels for fits done by `AGAUS` and `ZAMAN`. Data are in yellow, models in green, and residuals in pink.

enter into the average of that parameter. Finally, the task plots a user-selected number of the parameters and, optionally, prints all of them to a text file. The plot may appear on the TV or be placed in a standard plot file attached to the I polarization image.

Optionally, XG2PL will also add the spectrum of the V polarization data and the results of the fitting done by ZEMAN or ZAMAN at the same pixels as the I polarization Gaussians. The task reads the V polarization image for the data spectrum and the ZE table for the Zeeman-splitting model (either using the Gaussians or the simpler ones using the I polarization data). It computes the spectrum of each component (including the gain term in each), the net model (sum of the components but including the gain term only once), and the residual. Finally, the task plots a user-selected number of the parameters and, optionally, prints all of them to a text file. In general, the I polarization spectrum appears in the upper part of the plot and the V polarization spectrum appears in the lower portion. Either portion may be omitted under control of the adverbs. The output of XG2PL is illustrated in Figure 14 for the absorption model at each "source".

The inputs for XG2PL begin with the I polarization image which is required and then the V polarization image which may be omitted. INVERS and IN2VERS give the version numbers of the XG and ZE tables, respectively. APARM provides the central pixel coordinates, the plot intensity ranges for I and for V, the size of the rectangle (or circle) over which to average, and a flag limiting the average to those pixels having a model fit, BPARM is a set of flags selecting which parameters are plotted. CPARM selects the channel range to be plotted, the relative size of the V and I plots, the type of the horizontal axis (channels, frequency, velocity), whether the channels are plotted in reverse order, and whether the data are plotted with stepped or directly connected lines. For absorption models, CPARM also offers the option of plotting the I spectra as optical depth rather than as observed. OUTTEXT specifies the output text file, if any. The usual XYRATIO, LTYPE, DOTV, and GRCHAN adverbs control the scale, labeling, choice of TV versus plot file, and, if TV, which graphic channel(s) are used. If GRCHAN is zero, graphics channel 1 is used for data and labeling (usually yellow), channel 2 is used for the full model (usually green), channel is used for the residual (usually pink), and channel 4 is used for the model component(s) (usually cyan).

5 Image model creation

AIPS Memo 118 describes a wide variety of modeling tasks which will not be discussed here. A new, simplified task was written to provide direct studies of the fitting in XGAUS, AGAUS, ZEMAN, and ZAMAN.

The new task is called MODAB and has a small number of adverbs. COORDINA, CELLSIZE, APARM(1), and APARM(2) set image header parameters with values that are not otherwise used. IMSIZE and CPARM(3) set the size of the image cube in pixels. APARM(4) through APARM(9) define a Gaussian which is the continuum "source" when imaging in absorption or is used to scale the emission as a function of position when making an emission cube. APARM(10) defines the fractional leakage of the I polarization into V and FLUX sets the noise level in Jy/beam. OPTYPE = 'EMIT' requests an emission cube, otherwise an absorption cube is produced.

A text file, pointed at with INLIST contains up to 99 spectral features to be spread over the cube. Each line must contain

1.	Central optical depth (unitless) or peak brightness (Jy/beam)
2.	Central spectral channel (pixels)
3.	Full width of line at half-maximum (pixels)
4.	Separation of R and L polarization in channels (pixels)

6 Systematic errors

The image model task described above was used in both emission and absorption with high signal-to-noise to check for "systematic errors" in the answers returned by XGAUS, AGAUS, ZEMAN and ZAMAN. The I

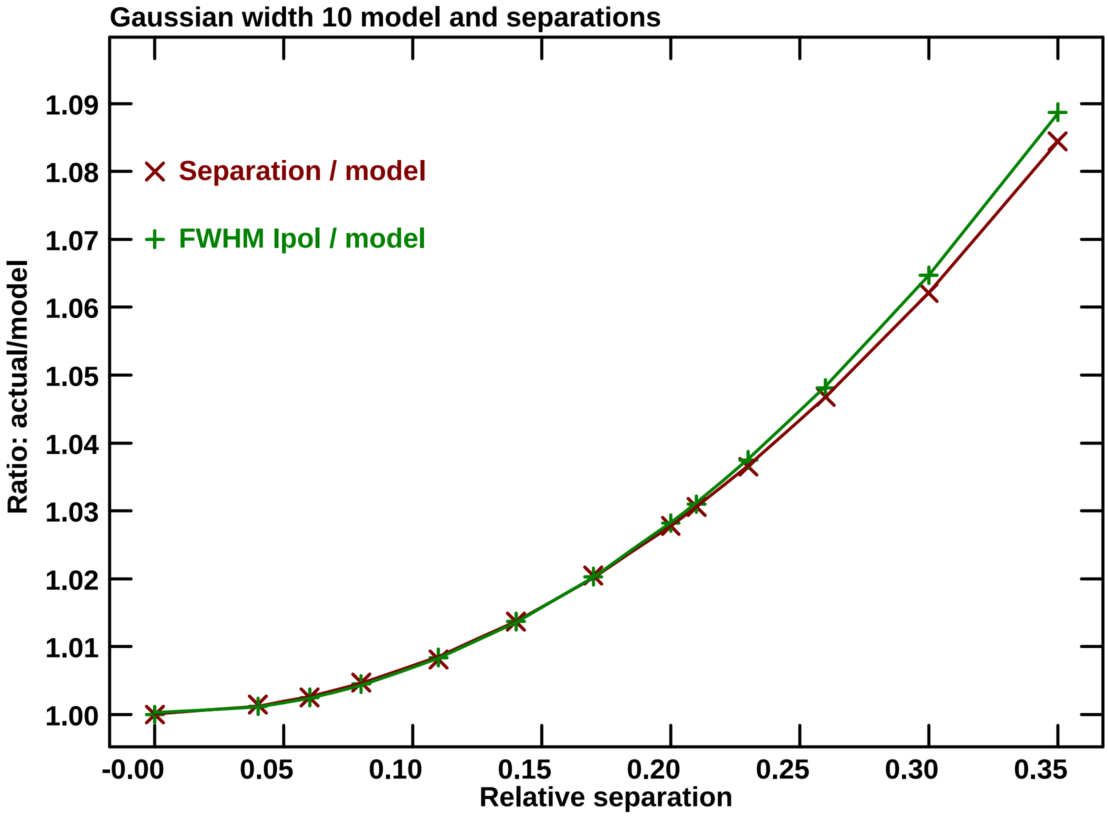


Figure 15: Measured relative widths (s/σ) in green and splittings (s/σ) in red plotted against the model splitting δ/σ.

polarization Gaussian width, not surprisingly, is measured to be wider than σ_i since the emission profile is widened by the splitting due to the magnetic field. The quality of the fit to a Gaussian also deteriorates as the emission line becomes less Gaussian in shape when the splitting becomes significant with respect to the unsplit line width. The measured splitting in V polarization is computed via Equation 17 and so would appear to suffer from the bias in the measured σ_i squared. Experimentation with the modeling tasks shows, however, that the bias in the splitting is almost the same (fractionally) as in the width. A variety of model separations (δ) were tried at several fixed component widths (σ) with very good signal-to-noise. The apparent Gaussian full-widths at half maximum (s) were measured with `XGAUS` and the apparent Zeeman splittings (d) were measured with `ZEMAN`. When plotted as relative widths (s/σ) and relative separations (d/σ) against δ/σ, as in Figure 15, the curves for the tested values of σ were approximately the same. The plotted fits to the curves in Figure 15 are given by

$$s/\sigma = 1.0 - 0.010(\delta/\sigma) + 0.7485(\delta/\sigma)^2 \qquad (19)$$

$$d/\sigma = 1.0 - 0.002(\delta/\sigma) + 0.6827(\delta/\sigma)^2 \qquad (20)$$

with an rms of the fit of about 0.0002.

Note a certain difficulty with these formulæ. Users of `XGAUS`, `AGAUS`, `ZEMAN` and `ZAMAN` will obtain values of s and d and will have to find δ and σ by some inverse method. Note too that, while doing these tests, some rather abnormal values were found, especially for d, despite the rather good signal-to-noise. It may be advisable to use the `MODAB` modeling task to simulate your data in order to determine the real uncertainties and systematic errors which apply.